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Modeling and analysis of planar symmetric superelastic flexure hinges

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ABSTRACT

Flexure hinges are frequently used in compliant mechanisms to obtain micro movements with high precision. In this paper, the planar symmetric superelastic flexure hinge is proposed. The motion capacity of the superelastic flexure hinge is significantly increased compared with conventional flexure hinges due to the distinguished features of the superelastic materials. The proposed flexure hinge is modeled by beam elements which consider the variation of the beam cross-section, and the geometric and material nonlinearities. Based on that model, the static responses of the planar symmetric superelastic flexure hinges with different notches are compared and analyzed. Both of the numerical calculation and the experiments indicate that the proposed methodology can accurately predict the deformation of the superelastic flexure hinges, and it also effectively decreases the calculation cost compared with FEA by ANSYS. In addition, three indexes are proposed and defined to evaluate the performance of the superelastic flexure hinge and the influence of the geometric parameters and the notch shapes on the performance of the hinge are also investigated.

1. Introduction

Flexure hinges are designed to deliver smooth motion between two adjacent rigid components in compliant mechanisms. Different from conventional rotary joints which are constructed by several different members, flexure hinges obtain their mobility by the deformation of themselves. The monolithic structure can overcome some inherent shortcomings existing in conventional mechanical systems such as friction, wear, and backlash (Paros and Weisbord, 1965; Yong et al., 2008). Therefore, compliant mechanisms integrated with flexure hinges are capable of being employed in the applications where high precision are required, e.g., micro positioning stages, high-precision alignment devices, micro manipulators, scanning electron microscopies and high-precision antennas (Lobontiu and Cullin, 2013; Du et al., 2014; Bhagat et al., 2014).

Planar notched flexure hinges are usually fabricated by removing two symmetric cutouts from the rectangular strips, as shown in Fig. 1. The main disadvantage of those flexure hinges is their limited motion range due to stress concentration, which severely restricts the applications of flexure hinges and compliant mechanisms (Ahuett-Garza et al., 2014). One of the most popular ways to enlarge the mobility of the flexure hinges is to increase the effective deformation length, e.g., the filleted leaf hinge shown in Fig. 2, which is able to deform all along the beam (Tian et al., 2010; Kim et al., 2012). However, the motion errors of this kind of flexure hinges are much larger than notched flexure hinges due to the rotation center drift. Another alternative approach is to employ superelastic materials which provide larger allowable strain to fabricate flexure hinges. Spring steels are always chosen to fabricate flexure hinges in the literatures for their excellent elasticity, but they are not the best candidates (Kern et al., 2013). The maximum elastic strain of the spring steels is 0.2–0.4%, while that for superelastic materials, e.g., shape memory alloy (SMA) is 8% (Desroches and Delemont, 2002). Obviously, the mobility of a flexure hinge made of superelastic material can be greatly improved, and the deviation between a superelastic flexure hinge and an ideal rotational joint is also acceptable (Hesselbach and Raatz, 2000).

In this paper, we study the static deformation of planar symmetric flexure hinges made of superelastic materials. Based on the small deformation assumption and the linear elastic beam theory, various methods have been proposed to model the deformation of conventional flexure hinges, including the integration of beam theory (Wu and Zhou, 2002), Castigliano's second theorem (Lobontiu et al., 2002; Shi et al., 2013), the equivalent beam methodology (Zettl et al., 2005) and the linear finite element method (FEM) (Friedrich et al., 2014). However all of the methods mentioned above cannot be used directly for modeling the superelastic flexure hinge, because the deformation of the hinge is relatively large

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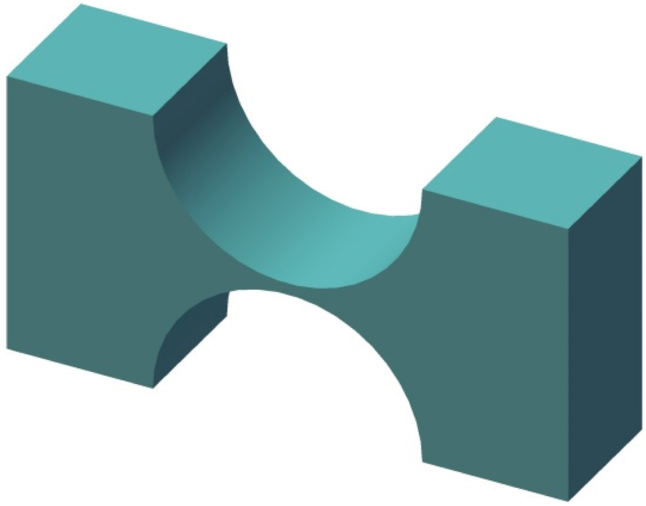

Fig. 1. Planar flexure hinge.

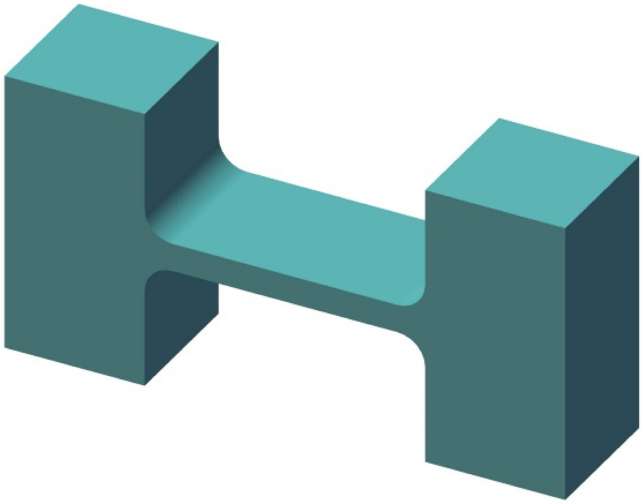

Fig. 2. Filleted leaf hinge.

and the Young's modulus of the material is also variable during the deformation process.

Formulating the deformation model for superelastic flexure hinges is a very challenging work. The variation of the cross-section, and the geometrical and material nonlinearities involved in this problem make it extremely complicated to obtain analytical solutions to describe the static responses of superelastic flexure hinges. Nonlinear FEM is probably the best way to solve such a problem. Although many nonlinear beam elements considering geometric and material nonlinearities have been proposed, they have been infrequently reported to be applied in the modeling and analyzing of the superelastic flexure hinges.

In this paper, the deformations of the planar symmetric superelastic flexure hinges are modeled by using nonlinear FEM. A non-prismatic co-rotational beam element presented in Battini and Pacoste (2002) and Pacoste and Eriksson (1997) is employed to establish the mapping between the load and the deformation of superelastic flexure hinges. In addition, three indexes based on the proposed model are defined to evaluate the performance of the superelastic flexure hinges and the influence of the geometric parameters and the notch shapes on the performance of the flexure hinges are also discussed.

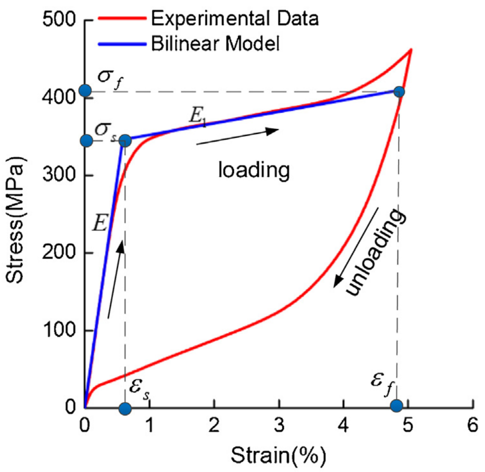

Fig. 3. Experimental data of Nitinol uniaxial tensile behavior and the bilinear constitutive model.

2. Flexure hinges based on the superelastic material

SMA is a typical superelastic material. It can conduct phase transformation between a high-ordered austenite phase and a low-ordered martensite phase under certain temperature and stress (Liew et al., 2004). The special microstructure brings SMAs two distinguished features, i.e., the shape memory effect and the superelasticity. The superelasticity allows the material undergoing very large strains and being fully recovered after unloading (Auricchio et al., 1997). Stress–strain curve of Nitinol (one of the most frequently used SMA materials) measured by uniaxial tensile test is presented in Fig. 3.

Many phenomenological constitutive models have been proposed to describe the unique mechanical behavior of SMAs. For simplicity, a bilinear constitutive model based on the experimental data is employed in this paper. As it is shown in Fig. 3, E is the Young's modulus of the material in the elastic stage, E_1 is the Young's modulus in the stage of phase transformation, ε_s and σ_s are the strain and stress at the start point of transformation respectively, and ε_f and σ_f are the strain and stress at the finial point of transformation. The relationship between stress and strain can be formulated as below.

$$\sigma = \begin{cases} E\varepsilon & (\varepsilon \le \varepsilon_s) \\ \sigma_s + E_1(\varepsilon - \varepsilon_s) & (\varepsilon > \varepsilon_s) \end{cases} \tag{1}$$

In order to obtain the corresponding material parameters, tensile tests were carried out with two samples and each sample performed 5 times. The material parameters obtained from experimental measurement are listed as below.

$$\begin{aligned} &E = 58.51\,\text{GPa} \quad E_1 = 1.5\,\text{GPa} \\ &\sigma_s = 346\ \text{MPa} \quad \sigma_f = 410\,\text{MPa} \end{aligned} \tag{2}$$

A sketch illustrating the notch profiles of the planar symmetric superelastic flexure hinges is shown in Fig. 4. In order to investigate the influence of the notch shapes on the performance of superelastic flexure hinges, the planar symmetric flexure hinges with three profiles, i.e., ellipse (E), parabola (P) and hyperbola (H) are discussed in this paper.

The parameters that define the flexure hinge's geometry are the notch length l, the minimum thickness t_s, the constant height h and the width b (not shown in the figure) of the Nitinol stripe. In this

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