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Isotropic springs based on parallel flexure stages

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ABSTRACT

We define *isotropic springs* to be central springs having the same restoring force in all directions. In previous work, we showed that isotropic springs can be advantageously applied to horological time bases since they can be used to eliminate the escapement mechanism [7]. This paper presents our designs based on planar serial 2-DOF linear isotropic springs. We propose two architectures, both based on parallel leaf springs, then evaluate their isotropy defect using firstly an analytic model, secondly finite element analysis and thirdly experimental data measured from physical prototypes. Using these results, we analyze the isotropy defect in terms of displacement, radial distance, angular separation, stiffness and linearity. Based on this analysis, we propose improved architectures stacking in parallel or in series duplicate copies of the original mechanisms rotated at specific angles to cancel isotropy defect. We show that using the mechanisms in pairs reduces isotropy defect by one to two orders of magnitude.

1. Introduction

The biggest improvement in timekeeper accuracy was due to the introduction of the oscillator as a time base, first the pendulum by Christiaan Huygens in 1656 [10], then the balance wheel–spiral spring by Huygens and Hooke in about 1675, and the tuning fork by Niaudet and Breguet in 1866 [14]. Since that time, these have been the only mechanical oscillators used in mechanical clocks and in all watches.

In [7], we presented new time bases for mechanical timekeepers which, in their simplest form, were based on a harmonic oscillator first described in 1687 by Isaac Newton in *Principia Mathematica* [13, Book I, Proposition X]. This oscillator is the isotropic harmonic oscillator, where a mass m at position $\boldsymbol{r}$ is subject to a central linear (Hooke) force.

Since the resulting trajectories have unidirectional rotation, this oscillator has the advantage of solving the problem of inefficiency of the escapement by eliminating it or, alternatively, simplifying it [7]. Isochronism is the key feature of a good time base, and in this case, the spring of the spring–mass system must be as isotropic as possible, meaning that in every direction, the spring stiffness and mass must remain the same. In addition, it should be planar in order to be easy to manufacture at any scale (note that Newton's model implies planar motion, by preservation of angular momentum).

In this paper, we mechanize Newton's model by designing new planar isotropic springs. Our designs are based on the principle of *compliant XY-stages* [1,3,11,12] which are mechanism with two degrees of freedom (2-DOF) both of which are translations. As these mechanisms are composed of compliant joints [9] they exhibit planar restoring forces so can be considered as planar springs. In the literature, many planar flexible *XY*-stages have been proposed and if some may be implicitly isotropic, none has been explicitly declared to be isotropic. This could be explained by the fact that, in general, *XY*-stages are controlled in closed-loops [17] and isotropy stiffness defects are therefore not necessarily a matter of concern. Moreover, we use a serial architecture instead of the parallel one generally seen in *XY*-stages used actuator integration applications.

Simon Henein [6, p. 156, 158] proposed two non-planar architecture *XY*-stages exhibiting planar isotropy. The first is composed of two serial compliant four-bar mechanisms, also called parallel arm linkage, which produce, for small displacements, translations in X and Y (see also [5]). The second is composed of four parallel arms linked by eight spherical joints and a bellow connecting the mobile platform to the ground.

In this paper the designs of two central springs based on parallel leaf springs are presented after a brief presentation of the context. For each of both designs, their analytical model is presented and compared to the performance based on finite element analysis and on experimental data of physically constructed prototypes.

Some images of these designs have appeared in [7] and some appear in recent patent applications.

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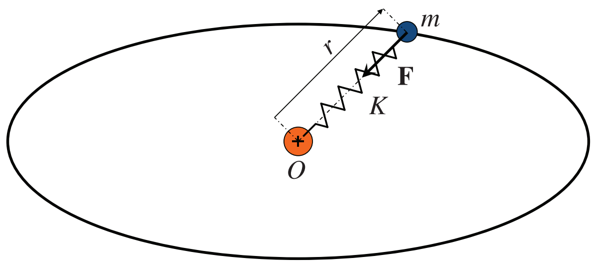

Fig. 1. Elliptical orbit under central Hooke Law.

2. Context

In order to understand the elliptical trajectories of planets predicted by Kepler's Laws, Isaac Newton considered the possible central laws producing elliptical orbits and he showed that apart from the inverse square law, a linear Hooke's law would also produce elliptical orbits,[1] see Fig. 1.

Newton's result is very easily shown. Consider a point mass moving in two dimensions subject to a central force

$$F(r) = -Kr,$$

where r is the distance of the mass to the center. Applying Newton's second law $F = ma$, where m is the mass of the particle and a its acceleration, gives the general solution

$$\mathbf{r} = (A_1 \sin(\omega_0 t + \varphi_1), A_2 \sin(\omega_0 t + \varphi_2)), \tag{1}$$

for initial conditions $A_1, A_2, \varphi_1, \varphi_2$ and frequency

$$\omega_0 = \sqrt{\frac{K}{m}}.$$

This shows that orbits are elliptical, but also that the period only depends on the mass m and the stiffness K of the central force, and not on the energy of the system, what is generally called *isochronism*. This last property is the key feature of horological time bases in which the regulation must be kept independent of the energy source. It follows that this oscillator is a good candidate to be a time base for a timekeeper, an observation first made in our previous article [7].

In order to exploit this oscillator as a mechanical time base, Newton's model must be followed as closely as possible. In particular, the mechanism's central linear restoring force must be as isotropic as possible. Expressed quantitatively, the isotropy defect must be minimized.

3. Definition of isotropy defect

The first step in analyzing the isotropy defect of a central spring is to give a precise definition of what is meant by *isotropy defect*. In particular, since isotropy defect only applies to central springs, the term "central" will be suppressed without ambiguity. The basic context of our isotropy defect computations is given in Fig. 2.

3.1. Baseline behavior

In order to evaluate isotropy defect, a baseline is required for comparison. We assume that our spring has *ideal stiffness* K. A force $\mathbf{F}_\theta$ of magnitude F and direction θ is applied, where θ will vary between 0° and 360° and the magnitude F will be constant (i.e.

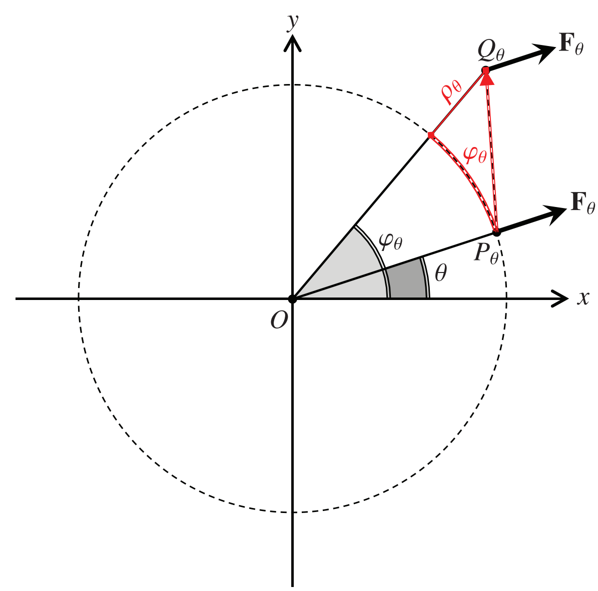

Fig. 2. Basic model of isotropy defect.

independent of θ). Under this force, the point O on the spring moves to the *ideal position* P_θ, and by Hooke's Law, $\vec{OP}_\theta$ has magnitude F/K and direction θ, see Fig. 2. Therefore, as θ varies between 0° and 360°, the ideal point P_θ describes a perfect circle of radius F/K, and the restoring force is linear and isotropic.

3.2. Definitions

The previous example illustrates ideal behavior with zero isotropy defect. Divergence from this example will be used to measure the isotropy defect. Thus, when isotropy defect does occur, the force $\mathbf{F}$ will move the point O to a point Q_θ generally distinct from P_θ, see Fig. 2. The *isotropy defect vector* in the direction θ is defined to be $\vec{P_\theta Q_\theta}$.

In order to evaluate and compare the isotropy defect of our mechanisms, it is more convenient to have scalar measures of isotropy defect. We therefore define the simpler *radial isotropy defect* given by

$$\rho_\theta = \|\vec{OQ}_\theta\| - \|\vec{OP}_\theta\|.$$

Note that this measure of isotropy defect considers the discrepancy between the magnitudes of the actual displacement and the ideal displacement for angle θ which is different from the magnitude $\|\vec{P_\theta Q_\theta}\|$ of the isotropy defect vector, see Fig. 2.

The *angular isotropy defect* φ_θ is defined as the angle between $\vec{OP}_\theta$ and $\vec{OQ}_\theta$, as measured in the counterclockwise direction, see Fig. 2.

In order to define stiffness isotropy defect, we first introduce the notion of stiffness in a given direction θ by

$$k_\theta = \frac{F}{\|\vec{OP}_\theta\| + \rho_\theta} = \frac{F}{\|\vec{OQ}_\theta\|}.$$

The *stiffness isotropy defect* in the θ direction is then

$$\Delta k_\theta = K - k_\theta,$$

where K is the ideal stiffness defined as the maximum of k_θ for all angles. Note that Δk_θ is non-negative, by definition of K.

The *relative stiffness isotropy defect* in the θ direction is defined as

$$\eta_\theta = \frac{\Delta k_\theta}{k_\theta} = \frac{\rho_\theta}{\|\vec{OP}_\theta\|},$$

[1] The occurrence of ellipses in both laws is now understood to be due to a relatively simple mathematically equivalence [4] and it is also well-known that these two cases are the only central force laws leading to closed orbits [2,15].