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PII: S0266-8920(17)30116-9
DOI: http://dx.doi.org/10.1016/j.probengmech.2017.07.005
Reference: PREM 2935

To appear in: *Probabilistic Engineering Mechanics*

Received date : 1 June 2017
Accepted date : 12 July 2017

A closed form approximation and error quantification for the response transition probability density function of a class of stochastic differential equations

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Abstract

A closed-form analytical approximation is derived for the response transition probability density function (PDF) of a certain class of stochastic differential equations with constant drift and nonlinear diffusion coefficients. This is done by resorting to a recently developed Wiener path integral based technique (WPI) in conjunction with a Cauchy-Schwarz inequality treatment of the problem. The derived approximation can be used, due to its analytical nature, as a direct SDE response PDF estimate that requires zero computational effort for its determination. Further, it facilitates an error quantification analysis, which yields an a priori estimate of the anticipated accuracy obtained by applying the approximate methodology. The reliability of the approximation is demonstrated via several engineering mechanics/dynamics related numerical examples pertaining to the stochastic beam bending problem, as well as to the response determination of stochastically excited nonlinear oscillators.

Keywords: Stochastic Differential Equations, Stochastic Dynamics, Path Integral, error quantification, Cauchy-Schwarz inequality.

1. Introduction

Monte Carlo simulation (MCS) schemes have been among the most versatile tools for solving stochastic differential equations (SDEs) of general form [1]. Nevertheless, in many cases they can be computationally prohibitive; and thus, there is a need for developing alternative approximate analytical/numerical solution techniques. Indicatively, in the field of stochastic engineering dynamics, techniques such as stochastic averaging [2, 3], statistical linearization [4–6], probability density evolution schemes [7], as well as methodologies based on Markov approximations and related Fokker-Planck equations [8] constitute potent alternatives to a computationally cumbersome MCS solution treatment of the problem.

Further, one of the promising techniques relates to the concept of path integral introduced, independently, by Wiener [9] and Feynman [10]. In general, the SDE solution joint transition probability density function (PDF) can be expressed as a Wiener path integral (WPI), or in other words, as a functional integral over the space of all possible paths. Note, however, that analytical evaluation of the WPI is a

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