Contents lists available at ScienceDirect
Probabilistic Engineering Mechanics
journal homepage: www.elsevier.com/locate/probengmech

Estimation of residuals for the homogenized solution of quasi-periodic media

F. Cluni *, V. Gusella
University of Perugia - Department of Civil and Environmental Engineering, Via G. Duranti, 93 - 06125 Perugia, Italy

A B S T R A C T

In this work we analyze the convergence of the elastic coefficients in terms of the residuals as the dimensions of the sample of a random two-dimensional medium vary. In particular, the case of the masonry material which can be considered a heterogeneous solid with two phases (stones or bricks and mortar) is taken into account. In particular, we consider a masonry which presents a quasi-periodic micro-structure. A procedure of numerical generation of wall portions has been developed which takes into account not only the scale ratio but also the mechanical ratio and the geometrical ratio. The convergence of residuals has been highlighted in terms of probability density function and statistical moments, up to the second order, of the stiffness coefficients and of the log-Euclidean distance between the masonry samples and the closest isotropic material.

1. Introduction

Several materials are characterized by a non homogeneous micro-structure and an anisotropic mechanical behavior. The differential equations that rule the problem of the mechanical response can be expressed using a two scales approach; the first scale can be associated to the microscopic characteristics while the second is associated with the macroscopic ones. The homogenization is a very useful tools to describe the macroscopic behavior. In case of material with random micro-structure, the characteristics of the equivalent homogeneous material can be found by means of the Representative Volume Element (RVE). A special case is the *masonry* that can be regarded as an anisotropic heterogeneous material (composed by two phase: stone or bricks and mortar) with a *quasi-periodic micro-structure*. The interest in this particular typology of masonry is given by the fact that it can be found in several buildings of historical interest. Several researchers have investigated this kind of masonry. Zeman and Šejnoha [1] proposed a Statistical Equivalent Periodic Unit Cell (SEPUC) in order to study a random arrangement of phases in an heterogeneous material. A procedure to characterize stochastically the elastic moduli of irregular masonries has been presented in [2]. Cecchi and Sab [3,4] have proposed an homogenization procedure based on a Love–Kirchhoff model, which they applied to a quasi-periodic masonry obtained by perturbation of a running bond texture. Milani and Lourenço [5] proposed a limit analysis approach to characterize the response of a masonry made of randomly assembled blocks; then they used that approach in studying a real building made of quasi-periodic masonry [6]. The use of a RVE to assess the mechanical characteristics of a masonry with an irregular texture have been used in [7], where the result where compared with the outcomes of experimental tests. Finally, Mukhopadhyay and Adhikari [8,9] considered a random media with an honeycomb texture for which they derived closed-form expressions of elastic moduli, and they investigated the effect of the variation of geometrical characteristics on the overall response. The Authors proposed an homogenization approach in [10–12] which allows to estimate the RVE and to obtain the mechanical characteristics of the equivalent homogeneous material. Based on the approach by means the SEPUC, an alternative method, which takes into account the quasi periodic micro-structure has been proposed in [13]

When dealing with the RVE, an aspect to be considered is the difference between the mechanical characteristics of the sample of the non-periodic masonry and the mechanical characteristics of the homogeneous media which is assumed to be equivalent to the masonry. This difference is named *residual*, and its estimation in the case of quasi–periodic masonry is the subject of the present paper. In other terms, the residual can be used to measure the relation between the dimension of the selected RVE and the errors in estimation of equivalent mechanical characteristics.

This aspect has received a very limited attention in literature. Let us consider the elastic behavior of heterogeneous media with random texture described by an elliptic differential operators with random coefficient depending on a small parameter. Under adequate conditions [14–16], it is known that, as the parameter scale $\varepsilon \to 0$, the operator converge to an *averaged operator* with non random coefficients; this type of convergence is known as *G-convergence*. The accuracy of the approximation has been studied [17–19] but it is very difficult to adapt

* Corresponding author.
E-mail addresses: federico.cluni@unipg.it (F. Cluni), vittorio.gusella@unipg.it (V. Gusella).

https://doi.org/10.1016/j.probengmech.2017.09.001
Received 6 June 2017; Received in revised form 11 September 2017; Accepted 20 September 2017
Available online xxxx
0266-8920/

the proposed methods to masonry which is characterized by a quasi-periodic texture. In order to investigate this problem, the Authors, in previous papers [20,21], dealt with the case of the beam with Young's modulus randomly varying along the axis. In the present work, the approach is extended to the case of a two-dimensional continuum with randomly varying mechanical characteristics.

The convergence of the homogenized solution is analyzed in terms of the parameters which characterize the microscopic scale (ε, ratio of elastic moduli, concentration ratio), by means of numerical simulations. The obtained results permits a preliminary estimation of the size of the RVE that assures a prefixed value of the error.

In details, in Section 2 the definition of quasi-periodic masonry is recalled, and the procedure to generate random samples of the texture, depending on three previously introduced ratios (length scale, geometrical and mechanical), is briefly given. In Section 3 the results of the parametric analysis are presented in terms of component of stiffness tensor. In Section 4 the convergence is analyzed by means of different distances between stiffness tensors. In Section 5 the closest homogeneous isotropic material is introduced. Eventually, concluding remarks are reported Section 6.

2. Numerical generation of quasi-periodic masonry samples

When we consider the masonries of historical building, such as the two examples found in central Italy shown in Fig. 1, it is evident that the micro-structure of this material is not *completely random* since some amount of regularity can be observed. The stones have different dimensions (length and height), but those of roughly the same height are laid out to form horizontal rows. As a consequence, different rows have different height, nevertheless, the height of the stones of the same row have small oscillation around the mean height for that specific row.

Moreover, it should be noted that the length of the stones is also variable and the length-to-height ratio can assumes very different values (see Fig. 1); taking into account this observation, it is assumed, in what follows, that the length of the stones are uncorrelated with their height. Therefore, a quasi-periodic masonry arrangement of bricks/stones and mortar is obtained in such a way that horizontal stones of roughly the same height can be identified in the same row of the wall. Regarding the mortar joints, they have roughly the same thickness and the vertical ones are not aligned when two adjacent rows are considered.

To assess the influence of both the mechanical and geometrical parameters of the constituent phases on the response of the masonry and homogenization residuals, a parametric analysis has been performed.

As illustrated before, the fundamental characteristics of a quasi-periodic masonry are:

(i) The masonry wall is made by courses (rows) of stones;
(ii) It is possible to individuate continuous "bed joints"(or horizontal joints) of mortar, that is the length of every horizontal joints is equal to the width of the wall;
(iii) The "head joints" (or vertical joints) of two adjacent courses are not aligned.

The samples of quasi-periodic masonry are characterized by N rows with N stone (or $N+1$ stones, but in this latter case the first and the last are half stones really) and have a L_W overall length. In order to generate the samples, the following parameters have been chosen:

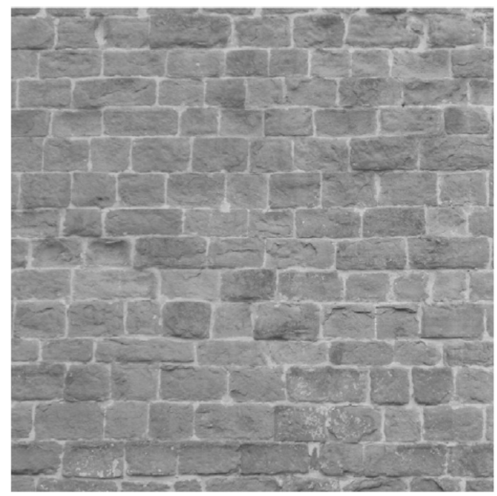
- the length scale ratio $\varepsilon = \bar{L}_b/L_W$, where $\bar{L}_b$ is the mean length of the stones;

- the geometrical ratio $g_r = \Delta L_b/\bar{L}_b = \Delta H_{br}/\bar{H}_b$, where $2\Delta L_b$ is the variation interval of the length of the stones, $\bar{H}_b$ and $2\Delta H_{br}$ are the mean and variation interval of the height of the stones;

- the mechanical ratio $m_r = E_b/E_m$ between the elastic moduli of stone and mortar (the Poisson's coefficient, both for stone and mortar, has been assumed equal to 0.2).

Given the previous conditions, the length ratio is $\varepsilon = 1/N$.

(a)



(b)

Fig. 1. Samples of quasi-periodic masonries.

Given ΔL_b and ΔH_{br}, the length and height of the stones have been assumed uncorrelated and randomly varying according to a uniform law around the mean values. The same hypothesis have been assumed to generate the thickness of head mortar joints t_h and bed mortar joints t_b, around the means $\bar{t}_h$ and $\bar{t}_d$.

Considering the ith course, the length of the jth stone of the course is given by

$$L_{b,ij} = \bar{L}_b + U_j \Delta L_b$$

where U_j indicates the jth number generated from the uniform law in the interval $[-1/2, 1/2]$. The *characteristic height* of the ith row is given by

$$H^i_{br} = \bar{H}_b + U_i \Delta H_{br}$$

while the height of the jth stone is obtained by

$$H_{b,ij} = H^i_{br} + U_j \frac{\bar{t}_b}{3}$$

and its position is determined by translating its base by the quantity

$$d_{ij} = U_j \frac{\bar{t}_b}{3}$$

Please cite this article in press as: F. Cluni, V. Gusella, Estimation of residuals for the homogenized solution of quasi-periodic media, Probabilistic Engineering Mechanics (2017), https://doi.org/10.1016/j.probengmech.2017.09.001.