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Metamodels of optimal quality for stochastic structural optimization

Christian Bucher
Center of Mechanics and Structural Dynamics, Technische Universität Wien, Karlsplatz 13, A-1040 Vienna, Austria

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ABSTRACT

Metamodels or response surface models are frequently chosen to reduce computational cost for structural optimization problems. These methods are also very popular for structural reliability analysis. It is therefore not surprising that response surface models are very attractive for reliability-based structural optimization. The paper discusses strategies to obtain a suitable response surface model, to assess its quality concerning prediction, and to use the response surface mode to identify important and unimportant variables. Several mathematical and structural examples illustrate the applicability of the presented approach.

1. Introduction

Due to the ever increasing demand on performance and cost-efficacy of structures, the need for numerical tools to optimize such structures in the design process has become very strong. The computational demand arising from optimization methods is quite heavy, and it is even more increased since various stochastic uncertainties have to be taken into account in the design optimization process (see e.g. [1]).

The sources of uncertainties in structural optimization may arise from several sources:

- Design variables (e.g. manufacturing tolerances)
- Objective function (e.g. tolerances, external factors)
- Constraints (e.g. tolerances, external factors)

These sources are indicated in Fig. 1.

For structural safety issues, the main concern are uncertainties in the constraints. This usually comes from the uncertainties in the loads (e.g. wind or earthquakes). The traditional design approach to take these unavoidable uncertainties into account is the introduction of so-called *safety margins* (cf. Fig. 2).

One of the major issues in establishing design procedures in code format is the appropriate definition of the safety margins. The rationale behind the choice of safety margins is to achieve a balance between safety and cost. It is well-known that high levels of safety in structures are associated with high initial cost, whereas low levels of safety lead to large expected failure cost. Assuming that both types of cost may be expressed in monetary terms, the appropriate level of safety (or reliability) is the one which minimizes the total expected cost including initial cost C_i and cost of failure C_f (see Fig. 3). Formulations for several optimization approaches involving risk and reliability are given e.g. in [2].

It can be seen from Fig. 3 that structural reliability (or conversely, the probability of failure) plays a central role as the governing parameter in cost-optimal probabilistic design. It is therefore of utmost importance to establish computational methods which allow the repeated, accurate computation of structural reliability within an iterative optimization procedure.

An optimization process in which reliability enters the objective or constraint functions is usually called *Reliability-Based Design Optimization* (RBDO) [1,3]. A typical computational flow for RBDO is sketched in Fig. 4. This flow is controlled by the optimization algorithm (e.g. a gradient-based method) which drives the design variables in the outer loop. The reliability constraint is then evaluated for each design, which in turn requires repeated structural analysis, e.g. for the First-Order Reliability Method (FORM). For several reliability constraints, the inner loops may have to be performed several times, i.e. separately for each constraint.

Assuming that the inner loop (for reliability analysis takes about 100 function evaluations) and that the outer loop needs about 100 different design evaluations, the total number of structural analysis would be $100 \times 100 = 10.000$. Given the current state-of-the-art of structural modeling this direct approach implies several thousands of expensive runs of very large Finite-Element models, hence in most cases this will not be not computationally feasible. Alternative approaches aim reducing the number of actual Finite-Element analyses to a minimum, typically for pre-selected combinations of the optimization variables. This is combined with a systematic variation of the random variables as well. Together, the computed samples of the relevant structural responses are utilized to build so-called *Response Surfaces*. The purpose of

E-mail address: christian.bucher@tuwien.ac.at.

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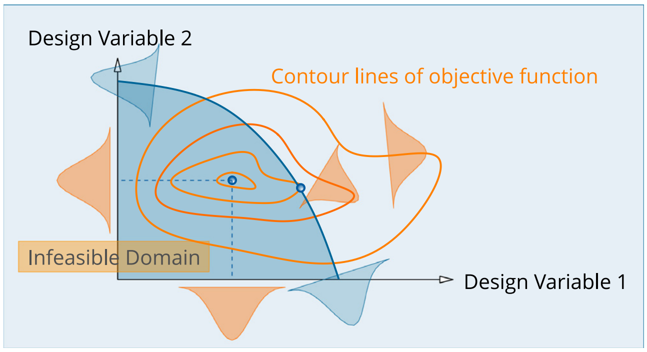

Fig. 1. Uncertainties in optimization.

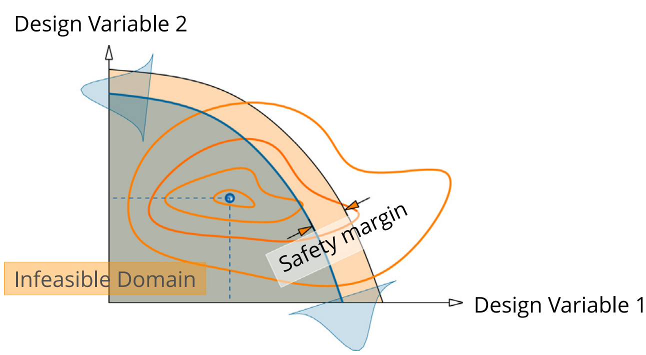

Fig. 2. Safety margin in optimization.

these Response Surfaces is to allow rapid computation of the structural responses for parameter combinations not analyzed beforehand [4,5]. For that purpose, the Response Surfaces must be simple mathematical functions whose describing parameters can easily be computed from the few actual Finite-Element results. Of course, this establishes an approximation procedure whose validity needs to be checked for each application.

2. Response surface models

2.1. Basic concept

As stated above, the basic idea of the Response Surface Method is to establish a continuous and easily computable function representing a possibly high-dimensional dependency for a specified response quantity on several input variables. For computational efficiency, this response

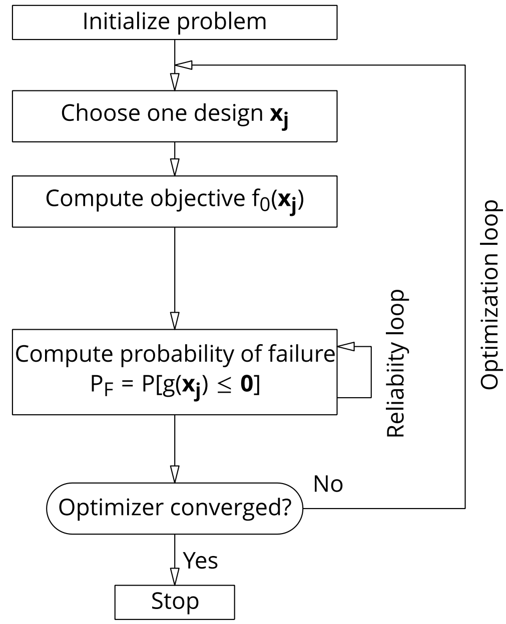

Fig. 4. Typical RBDO flow.

surface should be based on a sparse set of actual data points in which the exact relation between input and response is given. Essential concepts have been developed over a long period [6–10]. Applications in reliability analysis have been discussed in [11–18].

2.2. Mathematical formulation

The mathematical formulation for response surfaces is closely related to linear regression and interpolation modeling. Response surface model is based on linear regression if its functional form if linear in the unknown parameters p_k, i.e.

$$\eta(\mathbf{x}) = \sum_{k=1}^{n} p_k f_k(\mathbf{x}) \tag{1}$$

Fig. 5 schematically shows a regression response surface $\eta(x_1, x_2)$ [19].

The regression model is constructed from a sequence of input values $\mathbf{x}_i, i = 1 \dots m$ and corresponding model output values $y_i, i = 1 \dots m$. The set of parameters p_k can be determined by solving the least squares problem

$$S^2 = \sum_{i=1}^{m} \left[y_i - \eta(\mathbf{x}_i)\right]^2 \rightarrow \text{Min.!} \tag{2}$$

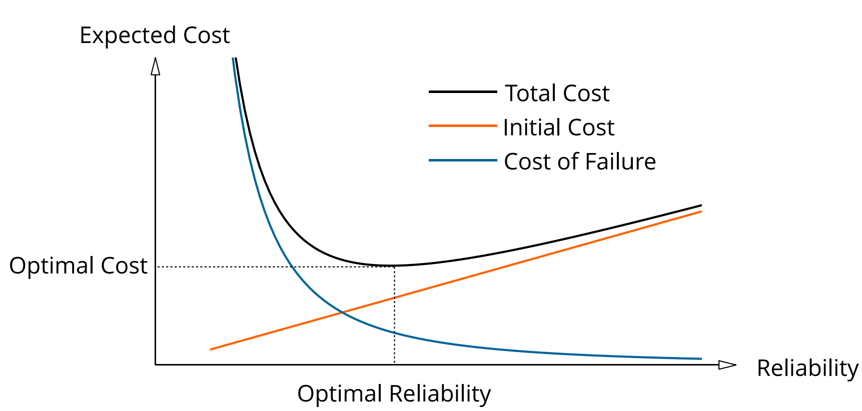

Fig. 3. Total expected cost depending on reliability.

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