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A new hybrid uncertainty analysis method and its application to squeal analysis with random and interval variables

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ABSTRACT

A new hybrid uncertainty analysis method with random and interval variables is proposed in this paper. In the proposed method, the uncertain parameters with sufficient information are treated as random variables, while the uncertain parameters with limited information are modeled as interval variables. Both random variables and interval variables can be viewed as special evidence variables. From this special point of view, both random variables and interval variables are represented by equivalent evidence variables in this study, and a unified framework for hybrid uncertainty analysis is developed based on evidence theory and subinterval perturbation technique. The effects of the mixture of random and interval uncertainties on uncertain output are assessed by belief measure and plausibility measure. On the base of the proposed method, the squeal analysis model of automotive disc brakes involving both random and interval uncertainties can be developed. A numerical example of brake squeal analysis is provided to illustrate the effectiveness of the proposed method. The analysis results show that the equivalent evidence variables can be explored to represent random variables and interval variables reasonably and the proposed method has good accuracy and efficiency in the hybrid uncertainty analysis of squeal instability The proposed method gives a unified framework to tackle several types of uncertain cases, and it is quite general and not only limited to the uncertainty analysis of brake squeal.

1. Introduction

As a complicated mechanical system, automotive disc brakes are subjected to a lot of uncertain factors. In brake systems, the uncertainties associated with friction, contact, wear, material properties and external loads are unavoidable [1] The noise, vibration and harshness (NVH) performance of brake systems is inevitably affected by uncertain factors. Brake squeal is a noise problem caused by the dynamic instability of frictional brake systems [2] and it has generated high warranty costs for car manufacturers Traditional numerical analyses of brake squeal are mainly based on deterministic techniques However, due to the effects of uncertainties, the squeal analysis results obtained through deterministic techniques will not well satisfy desired requirements or even become unfeasible Thus, quantifying the effects of uncertainties existing in brakes is of significance, sometimes even imperative in squeal analysis.

In the field of uncertainty analysis, random model is considered as the most useful technique to quantify uncertainties. In random model, the uncertain parameters are represented by random variables with accurate probability distributions [3] Based on random model, a variety of probabilistic methods have been developed, including Monte Carlo method (MCM) [4], perturbation stochastic method [5,6], polynomial chaos expansion method [7–9], etc Uncertainties widely exist in automotive brakes and some probabilistic methods have also been successfully applied into brake squeal analysis. For a comprehensive overview of the state-of-art on the probabilistic methods for brake squeal analysis interested readers are referred to [8–15]. In probabilistic studies a large number of statistical data is required to construct the precise probability distributions of uncertain parameters.

To cope with the uncertain case with limited information, non-probabilistic models have been developed Among the non-probabilistic models, interval model is most widely used, in which uncertainty is processed by interval variables with assigned lower and upper bounds. Based on the vertex method [16,17] and interval perturbation method [18,19] various non-probabilistic methods have been put forward to conduct interval analysis of real-world problems The overestimation of interval response due to dependency phenomenon is the main challenge

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in traditional interval studies. To reduce the overestimation a number of improved interval analysis methods have been proposed [20,21] Some attention has also been devoted to the comparison between the results of interval analysis and random analysis [22,23]. In the field of brake squeal analysis, an interval optimization approach [24] of squeal reduction has been proposed by the integration of interval model and genetic algorithm.

In practical engineering the complicated systems may involve both random and interval properties simultaneously. Thus, the hybrid uncertainty analysis model with random and interval variables has been developed With the help of hybrid uncertainty analysis model, significant successes have been achieved in the fields of reliability analysis [25,26] heat conduction analysis [27], acoustic response analysis [28,29] and vehicle dynamics analysis [30]. Recently, the squeal analysis and optimization of disc brakes with both random and interval variables have also been explored by integrating hybrid uncertainty analysis model, Monte Carlo simulation with reliability analysis [31,32] Although a large amount of research on hybrid uncertainty analysis is available, it should be noted that the existing methods [25–32] to resolve hybrid uncertainty problems with both random and interval variables are mainly based on the MCM, perturbation technique, random interval moment method, polynomial chaos theory or Chebyshev inclusion function.

In recent years, evidence theory [33] has received increasing attentions in uncertainty analysis Based on evidence theory, Du [34] and Hu et al. [35] have made attempts to handle uncertain problems with both probabilistic and evidence variables Evidence theory provides a quite flexible framework to quantify uncertainties [36], in the authors' opinion, evidence theory can be equivalent to random theory or interval theory under some special cases Thus, the aim of this paper is to extend evidence theory to solve the hybrid uncertainty analysis problems with both random and interval variables. The present study will make an attempt to put forward a new hybrid uncertainty analysis method and further apply it to the uncertain analysis of brake squeal.

In this study, the equivalent variables which are the special cases of evidence variables, are firstly explored to represent random variables or interval variables. Then, on the basis of equivalent variables, a new hybrid uncertainty analysis model is established, in which the effects of the mixture of random and interval uncertainties on uncertain output are assessed by belief measure and plausibility measure. Next, the belief measure and plausibility measure are efficiently computed with the aid of the combination of Taylor series expansion and subinterval analysis. Afterwards, the new analysis model is extended to brake squeal analysis involving both random and interval properties. Finally, a numerical example of brake squeal analysis is presented to illustrate the effectiveness of the proposed method The accuracy and efficiency of the proposed method are demonstrated by comparisons with the results provided by MCM.

2. Hybrid uncertainty analysis with random and interval variables

2.1. Random analysis

Let $\mathbf{a}^R = \{a_1^R, a_2^R, \dots, a_l^R\}^T$ denotes a random vector composed by l independent random variables The expectation $\mu(\mathbf{a}^R)$ and variance $\sigma^2(\mathbf{a}^R)$ of the random vector $\mathbf{a}^R$ can be expressed as

$$\mu(\mathbf{a}^R) = \mu(a_1^R, a_2^R, \dots, a_l^R), \quad \sigma^2(\mathbf{a}^R) = \sigma^2(a_1^R, a_2^R, \dots, a_l^R). \tag{1}$$

When the random variable a_i^R follows normal distribution, its distribution scope can be truncated to $[\mu(a_i^R) - \xi\sigma(a_i^R), \mu(a_i^R) + \xi\sigma(a_i^R)]$ $(i = 1, 2, \dots, l)$ approximately where ξ is a constant coefficient.

Let y denotes the analysis response of a given system For the system with only random variables $\mathbf{a}^R$, the system response can be expressed as $y = f(\mathbf{a}^R)$, where $f(\cdot)$ is the response function. Here y becomes a random variable due to the influence of $\mathbf{a}^R$ To calculate the random response of $y = f(\mathbf{a}^R)$, the MCM [4], stochastic perturbation method [5,6] and spectral stochastic method [8,9] have been widely adopted.

2.2. Interval analysis

Let $\mathbf{b}^I = \{b_1^I, b_2^I, \dots, b_n^I\}^T$ denotes an interval vector composed by n independent interval variables. $\mathbf{b}^I$ can be expressed as

$$\begin{aligned} &\mathbf{b}^I = [\underline{\mathbf{b}}, \bar{\mathbf{b}}] = \mathbf{b}^C + \Delta\mathbf{b}^I, \quad \Delta\mathbf{b}^I = [-\Delta\mathbf{b}, \Delta\mathbf{b}], \\ &\Delta\mathbf{b} = \frac{\bar{\mathbf{b}} - \underline{\mathbf{b}}}{2}, \quad \mathbf{b}^C = \frac{\underline{\mathbf{b}} + \bar{\mathbf{b}}}{2} \end{aligned} \tag{2}$$

where $\underline{\mathbf{b}}$ and $\bar{\mathbf{b}}$ represent the lower and upper bounds of $\mathbf{b}^I$, respectively; $\Delta\mathbf{b}$ and $\mathbf{b}^C$ represent the maximum deviation and the midpoint of $\mathbf{b}^I$, respectively; $\Delta\mathbf{b}^I$ denotes the deviation interval of $\mathbf{b}^I \cdot \mathbf{b}^I$ can also be expressed in component forms

$$\begin{aligned} &b_j^I = [\underline{b}_j, \bar{b}_j] = b_j^C + \Delta b_j^I, \quad \Delta b_j^I = [-\Delta b_j, \Delta b_j], \\ &\Delta b_j = \frac{\bar{b}_j - \underline{b}_j}{2}, \quad b_j^C = \frac{\underline{b}_j + \bar{b}_j}{2}, \ j = 1, 2, \dots, n \end{aligned} \tag{3}$$

where n is the number of interval variables. For the system with only interval variables $\mathbf{b}^I$, the system response can be expressed as $y = f(\mathbf{b}^I)$ Here, y becomes an interval variable due to the influence of $\mathbf{b}^I$. Several possible approaches have been developed to calculate the interval response of $y = f(\mathbf{b}^I)$, which include the MCM, numerical optimization methods, vertex method [16,17] and interval perturbation method [18,19], etc.

2.3. Hybrid uncertainty analysis with random and interval variables

In engineering practice, both the random and interval uncertainties may exist in a system simultaneously. For the given system with both random and interval variables, the system response can be expressed as

$$\begin{aligned} &y = f(\mathbf{a}^R, \mathbf{b}^I) \\ &\mathbf{a}^R = \{a_1^R, a_2^R, \dots, a_l^R\}^T \\ &\mathbf{b}^I = \{b_1^I, b_2^I, \dots, b_n^I\}^T. \end{aligned} \tag{4}$$

The above response function contains a l-dimensional random vector and a n-dimensional interval vector The response y has the characteristics of both random and interval variables, and it becomes an interval random variable.

To calculate the response of hybrid uncertainty problems with random and interval variables, several representative approaches are available in the literature. For examples, Gao et al. [25] and Xia et al. [28] have developed the hybrid perturbation Monte Carlo method and hybrid perturbation vertex method to calculate the expectations and variances of system responses Wu et al. [30] have delivered a Polynomial-Chaos–Chebyshev-Interval method to solve the vehicle dynamics problems involving hybrid random and interval variables Most of the existing researches involving both random and interval uncertainties, including the above studies, are mainly focused on the evaluation of the expectations and variances of system responses.

3. Hybrid uncertainty analysis based on evidence theory

Evidence theory provides a quite flexible framework to quantify uncertainty. Under some special cases, evidence variables can be equivalent to random variables or interval variables. In this section evidence theory will be extended to solve hybrid uncertainty analysis problems involving both random and interval properties.

3.1. Fundamentals of evidence theory

Evidence theory was firstly proposed by Dempster in 1976 [33] Some basic concepts of evidence theory are summarized as follows.

(1) *A Frame of discernment*

For an uncertain problem, evidence theory starts by defining a frame of discernment (FD), which consists of a set of mutually exclusive elementary propositions. For example, if a FD is given as $\mathbf{\Theta} = \{x_1, x_2\}$, where x_1 and x_2 are two mutually exclusive elementary propositions. $2^{\mathbf{\Theta}}$ is defined to denote the power set of $\mathbf{\Theta}$, which represents all possible