Probabilistic Engineering Mechanics 50 (2017) 1–8

Contents lists available at ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Full long-term extreme response analysis of marine structures using inverse FORM

Finn-Idar Grøtta Giske [a,b,*], Bernt Johan Leira [a], Ole Øiseth [c]

[a] Department of Marine Technology, NTNU, 7491 Trondheim, Norway
[b] Multiconsult, Nedre Skøyen vei 2, 0213 Oslo, Norway
[c] Department of Structural Engineering, NTNU, 7491 Trondheim, Norway

ARTICLE INFO

Keywords:
Marine structures
Extreme response
Long-term response
Stochastic processes
IFORM

ABSTRACT

An exact and an approximate formulation for the long-term extreme response of marine structures are discussed and compared. It is well known that the approximate formulation can be evaluated in a simplified way by using the first order reliability method (FORM), known for its computational efficiency. In this paper it is shown how this can be done for the exact formulation as well. Characteristic values of the long-term extreme response are calculated using inverse FORM (IFORM) for both formulations. A new method is proposed for the numerical solution of the IFORM problem, resolving some convergence issues of a well-established iteration algorithm. The proposed method is demonstrated for a single-degree-of-freedom (SDOF) example and the accuracy of the long-term extreme response approximations is investigated, revealing that the IFORM methods provide good estimates in a very efficient manner. The reduced number of required short-term response calculations provided by the IFORM methods is expected to make full long-term extreme response analysis feasible also for more complex systems.

1. Introduction

For the evaluation of extreme responses in the design of marine structures, a full long-term response analysis is recognized as the most accurate approach [1,2]. However, the computational effort is in many cases a limiting factor, and simplified approaches such as the environmental contour methods [3–5] are frequently used in practice. Over the last decade new methods have been proposed in an effort to make the full long-term approach more efficient, either by reducing the required number of short-term response calculations [2,6,7] or by computing the short-term quantities more efficiently [8–10]. In this paper we continue the development of robust and efficient methods for full long-term response analysis.

A comparison of different models for long-term extreme response can be found in [2]. In the present paper we focus on the models based on all short-term extreme peaks. For these models the long-term distribution of the short-term extreme value is formulated as an average of the short-term extreme value distributions weighted by the distribution of the environmental parameters. An exact formulation is obtained when an ergodic averaging is used, but using the population mean yields a very common approximate formulation.

In Section 2 of this paper we compare the exact and the approximate formulation, and show that the latter is non-conservative as it underestimates the long-term extreme responses. Nevertheless, the approximate formulation is commonly used because it readily lends itself to being solved very efficiently in an approximate manner by the first order reliability method (FORM) known from structural reliability. However, as we show in Section 3, the exact formulation can also be solved using FORM. To the authors' knowledge this has not been done before.

Section 4 deals with the numerical solution of characteristic values for the extreme response using inverse FORM (IFORM). IFORM was introduced in [3] for calculation of extreme response using environmental contours. The IFORM method has also been extended to a more general reliability context [11,12]. In [2] the IFORM solution for the extreme response of marine structures was found using a simple iteration algorithm proposed in [12]. This iteration algorithm has some convergence issues though, and these are addressed in the present paper. A new method is proposed for dealing with the convergence issues, using a sufficient increase condition along with a backtracking approach for the maximization problem being solved. It should be mentioned that an exact arc search algorithm [13] can also be used to obtain convergence,

* Corresponding author at: Multiconsult, Nedre Skøyen vei 2, 0213 Oslo, Norway.
E-mail address: finn.i.giske@ntnu.no (F.-I.G. Giske).

https://doi.org/10.1016/j.probengmech.2017.10.007
Received 19 April 2017; Received in revised form 11 August 2017; Accepted 18 October 2017
Available online 31 October 2017
0266-8920/

but this approach is expected to require a larger number of short-term response calculations. Furthermore, the proposed method is simpler in its form and will be easier to implement.

In Sections 5 and 6 a single-degree-of-freedom (SDOF) example is given, demonstrating the use of the proposed method. Some numerical results are also presented in order to compare the method with the standard iteration algorithm, and to assess the accuracy of the approximate formulation and the IFORM approximations.

2. Long-term extreme response modelling

For the assessment of long-term extreme responses of marine structures, it is common to model the environmental conditions as a sequence of short-term states during which the environmental processes are assumed stationary [1]. Each short-term state is defined by a collection of environmental parameters $\boldsymbol{S} = [S_1, S_2, \dots, S_n]$, with a joint probability density function (PDF) $f_{\boldsymbol{S}}(\boldsymbol{s})$ which we assume is given. We note that in order to be able to estimate $f_{\boldsymbol{S}}(\boldsymbol{s})$ in practice, an ergodicity assumption is required for the environmental parameters [14]. The long-term situation is composed of a large number N of short-term conditions, each of duration $\tilde{T}$, giving a long-term time duration of $T = N\tilde{T}$.

We denote by $\tilde{R}$ the largest peak of the response process during an arbitrary short-term condition, and by $\tilde{R}_{LT}$ the largest peak during the entire long-term period. Assuming that the short-term extreme values are independent, the long-term extreme value distribution $F_{\tilde{R}_{LT}}(r)$ is obtained as

$$F_{\tilde{R}_{LT}}(r) = F_{\tilde{R}}(r)^N, \tag{1}$$

where $F_{\tilde{R}}(r)$ is the cumulative distribution function (CDF) of the short-term extreme value $\tilde{R}$.

2.1. Formulations based on the short-term extreme peaks

Let the CDF of the largest peak during a short-term condition with environmental parameters $\boldsymbol{s}$ be given by $F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{s})$. The exact long-term CDF $F_{\tilde{R}}(r)$ of the short-term extreme value is obtained when an ergodic averaging is used [14,15], see also Section 12.4.2 of [1]. Thus we have the formulation

$$F_{\tilde{R}}(r) = \exp\left\{\int_{\boldsymbol{s}} \left(\ln F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{s})\right) f_{\boldsymbol{S}}(\boldsymbol{s})\, d\boldsymbol{s}\right\}. \tag{2}$$

The claim of exactness for the formulation (2) is perhaps somewhat unfortunate, since e.g. the assumption of stationary environmental processes is clearly not exact. The term "exact" is simply used here in the sense that the formulation (2) is the mathematically correct approach within the assumptions.

Usually, we are only interested in $F_{\tilde{R}}(r)$ for large values of r, which means that $F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{s}) \approx 1$. Using the linear approximations of the logarithm and the exponential function yields

$$\begin{aligned} F_{\tilde{R}}(r) &\approx \exp\left\{-\int_{\boldsymbol{s}} \left(1 - F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{s})\right) f_{\boldsymbol{S}}(\boldsymbol{s})\, d\boldsymbol{s}\right\} \\ &\approx 1 - \int_{\boldsymbol{s}} \left(1 - F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{s})\right) f_{\boldsymbol{S}}(\boldsymbol{s})\, d\boldsymbol{s}. \end{aligned}$$

From the properties of a PDF we know that the integral of $f_{\boldsymbol{S}}(\boldsymbol{s})$ over all values of $\boldsymbol{s}$ equals unity, and we obtain the approximation $F_{\tilde{R}}(r) \approx \bar{F}_{\tilde{R}}(r)$, where $\bar{F}_{\tilde{R}}(r)$ is the population mean

$$\bar{F}_{\tilde{R}}(r) = \int_{\boldsymbol{s}} F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{s}) f_{\boldsymbol{S}}(\boldsymbol{s})\, d\boldsymbol{s}. \tag{3}$$

The formulation (3) is a common approximation for the long-term CDF of the short-term extreme value, partly because it readily lends itself to being solved very efficiently by the FORM method. Furthermore, it is easy to mistakenly consider (3) as exact, because the formulation intuitively appears to be correct.

2.2. Connection with the average upcrossing rate formulation

If we assume that upcrossings of high levels are statistically independent, the short-term extreme peak distribution is given by

$$F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{s}) = \exp\left\{-\nu(r|\boldsymbol{s})\tilde{T}\right\}, \tag{4}$$

where $\nu(r|\boldsymbol{s})$ denotes the short-term mean frequency of r-upcrossings. For details we refer to Section 10.5 of [1]. Note that the expression (4) is only valid for high levels, i.e. for relatively large values of r. Inserting the expression (4) into (2) yields

$$F_{\tilde{R}}(r) = \exp\left\{-\tilde{T}\int_{\boldsymbol{s}} \nu(r|\boldsymbol{s}) f_{\boldsymbol{S}}(\boldsymbol{s})\, d\boldsymbol{s}\right\}, \tag{5}$$

and the relation (1) for the long-term extreme value distribution $F_{\tilde{R}_{LT}}(r)$ gives that

$$F_{\tilde{R}_{LT}}(r) = \exp\left\{-T\int_{\boldsymbol{s}} \nu(r|\boldsymbol{s}) f_{\boldsymbol{S}}(\boldsymbol{s})\, d\boldsymbol{s}\right\}, \tag{6}$$

where $T = N\tilde{T}$ is the long-term period. The expression (6) is also a common model for the long-term extreme response [14]. The fact that (2) and (6) are equivalent formulations is in agreement with what is found in [2].

2.3. Non-conservativity of the approximate formulation

As a simple consequence of Jensen's inequality, it can be show that $\bar{F}_{\tilde{R}}(r) > F_{\tilde{R}}(r)$. Indeed, since the natural logarithm is a strictly concave function, Jensen's inequality yields

$$\ln\left(E\left[F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{S})\right]\right) > E\left[\ln\left(F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{S})\right)\right],$$

where $E[\cdot]$ denotes the expectation operator. From (2) and (3) we realize that $\ln\left(F_{\tilde{R}}(r)\right) = E\left[\ln\left(F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{S})\right)\right]$ and $\bar{F}_{\tilde{R}}(r) = E\left[F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{S})\right]$, which means that $\ln\left(\bar{F}_{\tilde{R}}(r)\right) > \ln\left(F_{\tilde{R}}(r)\right)$ and hence $\bar{F}_{\tilde{R}}(r) > F_{\tilde{R}}(r)$.

From the result $\bar{F}_{\tilde{R}}(r) > F_{\tilde{R}}(r)$, it follows that exceedance probabilities will be smaller for the approximate formulation (3) compared to the exact formulation (2). This means that the formulation (3) will underestimate the long-term extreme values, making it a non-conservative approximation. Although the underestimation might not be significant, it is important to be aware of such an issue.

3. FORM formulations for long-term extremes

In this section we will show how the integrals of both formulations (2) and (3) can be solved in an approximate manner using the first order reliability method (FORM) found in connection with structural reliability analysis. In order to employ the FORM method, the formulations have to be rewritten in terms of a reliability problem. A reliability problem in the general sense is an integral written in the form

$$p_f = \int_{G(\boldsymbol{v})\le 0} f_{\boldsymbol{V}}(\boldsymbol{v})\, d\boldsymbol{v},$$

where $\boldsymbol{V}$ is a random vector with joint PDF $f_{\boldsymbol{V}}(\boldsymbol{v})$ [16]. Using reliability analysis terminology, the function $G(\boldsymbol{v})$ is referred to as the limit state function and the value of the integral p_f is called the failure probability.

3.1. Expressing the approximate formulation in terms of a reliability problem

That the integral (3) can be rewritten as a reliability problem, is well known. This is done by first rewriting

$$\bar{F}_{\tilde{R}}(r) = \int_{\boldsymbol{s}} F_{\tilde{R}|\boldsymbol{S}}(r|\boldsymbol{s}) f_{\boldsymbol{S}}(\boldsymbol{s})\, d\boldsymbol{s} = \int_{\boldsymbol{s}}\int_{\tilde{r}\le r} f_{\tilde{R}|\boldsymbol{S}}(\tilde{r}|\boldsymbol{s})\, d\tilde{r}\, f_{\boldsymbol{S}}(\boldsymbol{s})\, d\boldsymbol{s}.$$

We then define the random vector $\boldsymbol{V} = [\boldsymbol{S}, \tilde{R}]$, whose joint PDF will be $f_{\boldsymbol{V}}(\boldsymbol{v}) = f_{\tilde{R}|\boldsymbol{S}}(\tilde{r}|\boldsymbol{s}) f_{\boldsymbol{S}}(\boldsymbol{s})$. Thus we have

$$\bar{F}_{\tilde{R}}(r) = \int_{\tilde{r}\le r} f_{\boldsymbol{V}}(\boldsymbol{v})\, d\boldsymbol{v} = 1 - \int_{r-\tilde{r}\le 0} f_{\boldsymbol{V}}(\boldsymbol{v})\, d\boldsymbol{v},$$