Probabilistic Engineering Mechanics 50 (2017) 17–24

Contents lists available at ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Benchmark solutions of stationary random vibration for rectangular thin plate based on discrete analytical method

Guohai Chen [a], Jilei Zhou [b], Dixiong Yang [a,*]

[a] State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, International Research Center for Computational Mechanics, Dalian University of Technology, Dalian 116024, China

[b] School of Traffic and Vehicle Engineering, Shandong University of Technology, Zibo 255049, China

ARTICLE INFO

Keywords:
Stationary random vibration
Rectangular thin plate
Discrete analytical method
Benchmark solutions
Pseudo excitation method

ABSTRACT

This paper aims to accurately and efficiently achieve the benchmark solutions of stationary stochastic responses for rectangular thin plate. Firstly, the exact solutions of free vibration for thin plate with SSSS, SSSC, SCSC, SFSF, SSSF and SCSF boundary conditions are introduced to random vibration analysis. Based on pseudo excitation method (PEM), the analytical power spectral density (PSD) functions of the transverse deflection, velocity, acceleration and stress responses for thin plate under random base acceleration excitation are derived. Subsequently, to enhance computational efficiency, the discrete analytical method (DAM) that realizes the discretization for the modal coordinates and frequency domain is proposed. Finally, the efficiency of DAM and the accuracy of benchmark solutions are scrutinized by comparison with the analytical solutions and finite element solutions.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

As a basic structural member, the plate is widely applied to practical engineering. Usually, the plate structure is subjected to various excitations such as the earthquakes, winds, waves, turbulent boundary and jet noise, etc., which commonly present the randomness both in temporal and spatial domain. Random vibration analysis for a plate structure involves two types of model. The first is the continuous model based on the high-order partial differential equation, from which the analytical solution of random vibration response may be achieved. The second is the discrete model in which the continuum structure with infinite degrees of freedom is discretized to a multiple degrees of freedom (MDOF) system, by means of the numerical technique such as the popular finite element method (FEM). The discrete model can be utilized to approximately obtain the stochastic dynamical responses of structure. However, the continuous model can describe accurately its mechanical behavior, and is suitable to achieve the credible benchmark solutions of structures for verifying the discrete model and associated numerical methods. This work attempts to address the problem that there is a lack of benchmark solutions of random vibration responses, especially the stress solutions of thin plate.

In the past fifty years, the progresses on random vibration analysis based on the continuous model have been made. By virtue of normal mode method and the time domain Green function method, Lin [1] investigated the transient displacement responses for continuous structures subjected to stationary random excitations. Crandall and his colleagues [2,3] pointed out that, with exception for enhanced response in small zones and narrow lanes, the mean square velocity response of plate under stationary wide-band point random excitation presents uniform spatial distribution. Rosa and Franco [4,5] carried out the random vibration analysis for the rectangular thin plate subjected to the turbulent boundary layer excitation. However, only the simple supported edges for the beam or plate was tackled in their works. In fact, the boundary condition has considerable effect on the stochastic response of the structure, because its frequencies and mode shapes are completely different under different boundary conditions. Hosseinloo et al. [6] examined the effects of modal damping and excitation frequency range on the root mean square (rms) of acceleration response and the maximum deflection of thin plates with CCSS, SCCC, CCCC boundary conditions subjected to the base acceleration random excitation, and indicated that these responses decrease with the increasing of the modal damping ratio and the excitation frequency range. Nevertheless, it is difficult to achieve the benchmark solutions in their study, since the approximate frequencies and modal shapes were adopted. Moreover, the complete quadratic combination (CQC) based analytical approach might require

* Corresponding author.
E-mail address: yangdx@dlut.edu.cn (D. Yang).

https://doi.org/10.1016/j.probengmech.2017.10.006
Received 25 March 2017; Received in revised form 28 September 2017; Accepted 18 October 2017
Available online 28 October 2017
0266-8920/© 2017 Elsevier Ltd. All rights reserved.

the large computational efforts with the widening of frequency band of random excitation.

On the other hand, to balance the computational accuracy and efficiency, the significance of the modal cross-correlation was also intensively investigated. Crandall et al. [2,7–9] examined the effects of modal overlap ratio on the mean square response for various structures, and pointed out that the bandwidth of random excitation and the damping ratio of structures are the main influence factor on stochastic response, as well as suggested the modal-sum and image-sum approaches to decrease the approximate evaluation error of the sum of a large number of integral [10]. Elishakoff et al. [11–14] performed a series of researches on different structures such as curved panel and shell, to elaborate the dramatic effects of modal cross-correlation on the mean square responses. Meanwhile, the similar works for discrete MDOF systems were also conducted. In order to decrease the computational effort of stochastic dynamical analysis, an approximate square root of sum square (SRSS) method was applied by omitting modal cross-correlation terms [15]. Wilson et al. [16] developed a CQC method to reduce the numerical errors of SRSS rule, but at the expense of efficiency. Accordingly, Lin et al. [17–20] proposed a highly efficient and accurate algorithm named as pseudo excitation method (PEM), which promotes the engineering application of random vibration theory. The extensive application of the PEM is dependent on the development of finite element method. It is well known that the numerical errors of FEM will nonlinearly increase with the increasing frequency [21]. For obtaining the benchmark solutions, the analytical solutions of free vibration can be adopted to eliminate the errors in the band-wide random vibration analysis for the plate. In this paper, the benchmark solutions are achieved efficiently, when the plate is subjected to band-wide random excitation up to 20 kHz with the proposed PEM-based discrete analytical method.

For the free vibration analysis of plates, Leissa, Leissa and Qatu [22,23] reviewed the free vibration for rectangular thin plate with various boundary conditions, and pointed out that there are exact solutions of free vibration for only the 6 Lévy boundary conditions (SSSS, SSSC, SCSC, SFSF, SSSF, SCSF) with two opposite edges simply supported among 21 cases, which involve the possible combinations of clamped (C), simply-supported (S), and free edge (F) condition. Due to the difficulty of solving the fourth-order partial differential governing equation, the other 15 cases must be solved by the approximate approaches, such as FEM [24], finite difference method [25], finite strip method [26], boundary element method [27], differential quadrature method [28], Rayleigh–Ritz method [29], superposition method [30], symplectic superposition method [31,32] etc.

In this paper, the analytical PSD functions of stationary stochastic responses for rectangular thin plate with the 6 Lévy boundary conditions (SSSS, SSSC, SCSC, SFSF, SSSF and SCSF) are obtained. Therein, the exact solutions of free vibration of thin plate are introduced, and the pseudo excitation method based on the continuous model is employed. Through integrating the corresponding PSD functions, the rms of the displacement, velocity and acceleration responses as well as the stress components are achieved, whose results are also termed as benchmark solutions. Moreover, the discrete analytical method (DAM) is developed to improve computational efficiency without reducing the precision by discretizing the modal space coordinate and frequency domain.

The remainder of this paper is organized as follows. Section 2 revisits the exact Lévy solutions of free vibration for rectangular plate under 6 boundary conditions, which provides a foundation of benchmark solutions of stationary random responses. In Section 3, the analytical response PSD functions are derived by employing the PEM-based analytical method and discrete analytical method. Moreover, an approach to calculate the rms of stationary random response of thin plate is presented in Section 4. Then, two examples in Section 5 illustrate the benchmark solutions of stationary random vibration for the 6 cases under base wide-band white noise excitation and filtered white noise excitation. Comparison between the analytical solutions and finite element solutions verifies the high accuracy and efficiency of DAM. Section 6 draws some conclusions.

2. Exact solutions of free vibration of rectangular thin plate

2.1. Differential equation of forced vibration of rectangular thin plate

The differential equation of forced vibration for rectangular thin plate is given by [22,23]

$$D\nabla^4 w(x,y,t) + c\frac{\partial w(x,y,t)}{\partial t} + \rho h\frac{\partial^2 w(x,y,t)}{\partial t^2} = p(x,y,t) \tag{1}$$

where $D = Eh^3/12(1-\nu^2)$ is the bending rigidity of the plate; E the Young's modulus and υ the Poisson's ratio; $\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}$ is the bi-harmonic operator; $w(x, y, t)$ is the transverse deflection; c indicates the viscous damping coefficient of the plate; ρ is the volume density of the plate; h indicates the plate thickness; $p(x,y,t)$ is an excitation.

As shown in Fig. 1, the three classical boundary conditions for the rectangular plate, namely simply supported (S), clamped (C) and free (F) can be described by

$$\begin{aligned}
&\text{Simply supported (S)}: w = 0, \quad M_x = 0\\
&\text{Clamped (C)}: w = 0, \frac{\partial w}{\partial x} = 0\\
&\text{Free (F)}: M_x = 0, V_x = Q_x + \frac{\partial M_{xy}}{\partial y} = 0
\end{aligned} \tag{2}$$

where $\frac{\partial w}{\partial x}$ is the rotation angle in the xz plane; M_x denotes the bending moment in the xz plane; Q_x represents the shear force; M_{xy} is the torsional moment in the yz plane; V_x is the equivalent shear force.

2.2. Exact solutions of free vibration under 6 boundary conditions

The undamped free vibration differential equation of thin plate is formulated as [22]

$$D\nabla^4 w(x,y,t) + \rho h\frac{\partial^2 w(x,y,t)}{\partial t^2} = 0 \tag{3}$$

The transverse deflection in Eq. (3) is expressed as $w(x,y,t) = \phi(x,y)\exp(\mathrm{i}\omega t)$, in which ω indicates the angular frequency of free vibration, and $\phi(x,y)$ represents the corresponding modal shape. Substituting the transverse deflection formulation $w(x,y,t)$ into Eq. (3) can yield the differential equation about the modal shape function

$$D\nabla^4\phi(x,y) - \rho h\omega^2\phi(x,y) = 0 \tag{4}$$

For the 6 cases with a couple of simply supported boundary conditions on the opposite edge in Fig. 1(b), the Lévy solutions [22,23] of modal shape are expressed as

$$\phi(x,y) = \left(A_1\cos\lambda_1 y + A_2\sin\lambda_1 y + A_3\cosh\lambda_2 y + A_4\sinh\lambda_2 y\right)\sin\mu x \tag{5}$$

where $\mu = m\pi/a$, and m is the number of half-wave; a is the length of plate along the y coordinate; $\lambda_1 = \sqrt{\mu^2 - \sqrt{\rho h\omega^2/D}}$, and $\lambda_2 = \sqrt{\mu^2 + \sqrt{\rho h\omega^2/D}}$ can be calculated according to the corresponding frequency equations (see Table A in Appendix); $A_1 \sim A_4$ will be determined in terms of the boundary conditions.

Note that an assumption $\sqrt{\rho h\omega^2/D} > \mu^2$ is taken in Eq. (5), and frequency equations are listed in Table A. If $\sqrt{\rho h\omega^2/D} < \mu^2$, it must be replaced $\sin\lambda_1 y$ and $\cos\lambda_1 y$ with $\sinh\lambda_1 y$ and $\cosh\lambda_1 y$, respectively.

3. Discrete analytical method for stationary random responses

In this section, at first, the analytical procedure of stationary random vibration responses for rectangular thin plate is derived by combining the mode superposition method with the pseudo excitation method. Actually, it is not restricted by the form of random excitation, such as the point excitation, the distributed excitation or the base acceleration excitation.

Download English Version:

https://daneshyari.com/en/article/7180920

Download Persian Version:

https://daneshyari.com/article/7180920

Daneshyari.com