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On effects of track random irregularities on random vibrations of vehicle–track interactions

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ABSTRACT

As the integrated reflection of track substructure deformations and the most important excitation for vehicle–track interactions, track irregularities show random nature, and generally being regarded as weak stationary random processes. To better expose the full statistical characteristics of track random irregularities on amplitude and wavelength, a time–frequency transform and probability theory based model is developed to simulate representative and realistic track irregularity sets by combining with random sampling methods. Moreover, a three-dimensional (3-D) vehicle–track coupled model is established by finite element method and dynamic equilibrium equations, where the nonlinearity of wheel/rail interaction is considered. Finally, a probability density evolution method (PDEM) is introduced to solve the probabilistic transmission issues between track irregularity sets and dynamic responses of vehicle–track coupled systems. There is a clear demonstration that the results derived by proposed methods are comparable to the experimental measurements. Through effectively applying the above methodologies, the probabilistic and random characteristics of vehicle–track interaction can be properly revealed.

1. Introduction

Track irregularities, generally viewed as the rail profile deformations, are the integrated reflections of the track substructure deformations, and playing important roles in dynamic propagation mechanism of wheel/rail systems, wheel/rail noise, track maintenance, etc. It has been a common point of view that track irregularities possess random and evolutionary nature due to the wheel/rail cyclic interactions, subgrade settlement, material fatigues, etc. Obviously, the vehicle–track interactions excited by track irregularities will show great variability and stochasticity.

Until now, the railway dynamics with track irregularities considered are abundant, see for instance, Refs. [1–10]. While these researches mainly concentrate on typical track conditions, e.g., wheel flats, the specific forms of track profiles, etc., it is therefore insufficient that the full information of track irregularities on amplitude and wavelength measured from large-scale realistic railway lines has not been fully considered, and accordingly, the research results just represent the specific or local dynamic properties of vehicle–track interactions, and this type of analytical method will lose its power when solving some other scientific issues, e.g., reliability based track maintenance, stochastic vibrations, etc.

As stated above, though the randomicity of track irregularities has been involved in some studies, the probabilistic statistics on amplitude and wavelength of random track irregularities accompanied by a general approach to simulate them with high efficiency has not been fully developed. However, in some of the pioneering work, Balzer [11], Corbin [12] and Massel [13] started to use power spectral densities (PSDs) to express the stochastic irregularities of the track and model the random field of track irregularities, moreover, R.N. Iyengar and O.R. Jaiswal [14] reminded us of the non-Gaussian characteristic of irregularities measured on a sliding chord 3.6 or 9.6 m long and developed a non-Gaussian model to simulate the unevenness data. Later, they continuously proposed a random field model for estimating expected values of level crossing and peaks in a given track length [15]. Recently, some more developments have been achieved in data mining and stochastic modeling of track irregularities, for example, G. Perrin [16] devoted to a remarkable research on the stochastic modeling of the

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track geometry, which properly considered the statistical properties of this vector-valued, non-Gaussian and non-stationary track irregularity random field and most importantly took into account the dependencies of irregularities; and then, Alfonso et al. [17] used almost the same strategies to construct the stochastic model of track geometry irregularities.

Additionally, it is worth mentioning that more and more researchers began to pay attention to the random vibrations of railway dynamics, see for instance, Xu and Zhai [18] proposed a computational model to analyze the temporal–spatial stochastic vibrations of vehicle–track systems, in which the randomness of system parameters and excitations are effectively considered and coupled; Mao et al. [19] introduced the probability density evolution method (PDEM) into the random analysis of train–bridge coupled system involving random system parameters; Zhu et al. [20] and Zeng et al. [21] studies the stochastic vibration of vehicle–bridge system using pseudo-excitation method (PEM) by mainly considering the random characteristics of track irregularities.

Undoubtedly, the body of research represented above highlights the work done on the modeling of track irregularities and random vibrations of vehicle–track (bridge) systems. However, the foregoing researches, when used hand-in-hand with the probability theory must further be examined to clarify the probabilistic properties of track irregularities, since these properties are the foundation of the reliability-based parameters design and the dynamic analysis of the railway system. Further researches in this area are therefore recommended to promote the advancements of the random analysis of railway interaction systems.

The purpose of this paper is to identify available probabilistic information of track irregularities and to develop an inversion model for obtaining the representative track irregularity sets, shown in Section 2; then in Section 3, a vehicle–track coupled model (VTCM) are compiled for stochastic analysis of vehicle–track systems; moreover, by introducing the probability density evolution method (PDEM), the probabilistic transmission between track random irregularities and dynamic responses of vehicle–track systems is properly addressed, as presented in Section 4; finally, Section 5 validates the proposed method by comparing the calculated results with the actually measured ones and presents some typical responses of the vehicle systems and the track systems.

2. Track irregularity probabilistic model

In this paper, a track irregularity probabilistic model (TIPM) will be developed with the help of track irregularity PSD accompanied by the cumulative probability method. It is known that track irregularity power spectral densities at different wavelengths are randomly distributed with probabilistic properties.

Without loss of generality, the cumulative probability PSDs (CP-PSDs) are denoted as $\boldsymbol{\Psi}(\lambda,\boldsymbol{\omega})$, in which λ represents the vector of cumulative probabilities, $\boldsymbol{\omega}$ represents the spatial frequencies. $\boldsymbol{\Psi}(\lambda,\boldsymbol{\omega})$ can therefore be further expressed as

$$\boldsymbol{\Psi}(\lambda,\boldsymbol{\omega}) = \{\boldsymbol{P}_{\lambda_i,\omega_j} | \lambda_i \in [\lambda_l,\lambda_u], \boldsymbol{\omega}_i \in [\omega_l,\omega_u]\} \tag{1}$$

where $\boldsymbol{P}_{\lambda_i,\omega_j}$ denotes the power spectral densities holding for certain cumulative probability and spatial frequency, i.e., λ_i, $\boldsymbol{\omega}_j$. λ_l and λ_u denote the lower- and upper-limit of cumulative probabilities, ω_l and ω_u the lower and upper limits of spatial frequencies.

From Eq. (1), it is convenient to let $\boldsymbol{\Psi}(\lambda,\boldsymbol{\omega})$ take the form below

$$\boldsymbol{\Psi}(\lambda,\boldsymbol{\omega}) = \begin{bmatrix} \boldsymbol{P}_{\lambda_1,\omega_1} & \boldsymbol{P}_{\lambda_1,\omega_2} & \cdots & \boldsymbol{P}_{\lambda_1,\omega_m} \\ \boldsymbol{P}_{\lambda_2,\omega_1} & \boldsymbol{P}_{\lambda_2,\omega_2} & \cdots & \boldsymbol{P}_{\lambda_2,\omega_m} \\ \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{P}_{\lambda_n,\omega_1} & \boldsymbol{P}_{\lambda_n,\omega_2} & \cdots & \boldsymbol{P}_{\lambda_n,\omega_m} \end{bmatrix} \tag{2}$$

Previous study [22] has shown that power spectral densities for any arbitrary frequency obey specific probability distribution, from which it can be deduced that there are mathematical transferring relationships between spectral densities, spatial frequencies and cumulative probabilities, with noting that the $\boldsymbol{P}_{\lambda_i,\omega_j}$ can be solely confirmed by λ_i and $\boldsymbol{\omega}_j$ at a specified probability.

With the basic understanding presented above, one is capable of constructing the TIPM with the help of these relationships.

2.1. Calculation of track irregularity cumulative probability spectrum

In Ref. [22], it is pointed out that the spectral densities subjected to stationary random process of track irregularities will obey χ^2 distribution with 2 degrees of freedom (DOFs) after appropriate transformation, the conversion approach is shown by the following two steps:
(1) Start with the calculation of the mean value of track irregularity spectrum denoted as $\bar{\boldsymbol{S}}_k$, where k is corresponded to different frequencies;
(2) Conduct the transformation upon $\boldsymbol{S}_{k,j}$, which denotes a spectral density from the PSD sequence toward specific frequency point k, the transformation can be written as

$$\widetilde{\boldsymbol{S}}_{k,j} = 2\frac{\boldsymbol{S}_{k,j}}{\bar{\boldsymbol{S}}_k}, \quad j = 1,2,\ldots,N \tag{3}$$

in which N is the total number of spectral densities at the specific frequency k.

Consequently, one is reminded that $\widetilde{\boldsymbol{S}}_{k,j}$ obey χ^2 distribution with 2 DOFs defined by

$$f(x) = \begin{cases} \dfrac{1}{2^{\widetilde{n}/2}\Gamma(\widetilde{n}/2)} x^{\widetilde{n}/2-1}\mathrm{e}^{-x/2}, & x > 0 \\ 0 & \text{else} \end{cases} \tag{4}$$

where $\widetilde{n}$ denotes the number of DOFs, which is also the shape parameter, and $\Gamma(\widetilde{n}/2)$ denotes the gamma function.

Regarding track random irregularities as a weak stationary random process, the χ^2 distribution with 2 DOFs is generally appropriate in engineering application. But by precisely surveying the characters of χ^2 distribution with 2 DOFs, it can be observed that its shape curve is monotonically decreasing, which means that the probability distribution curves of spectrum densities are all strictly monotone, decreasing without coinciding with the measured results.

GEV distribution can be unified as the following formula [23], reads:

$$\mathrm{H}(x;\zeta,\mu,\sigma) = \exp\{-[1+\zeta(\frac{x-\mu}{\sigma})]^{-\frac{1}{\zeta}}\}\mathrm{I}(x) \tag{5}$$

in which μ and σ denote the location parameter and scale parameter, respectively; ζ denotes the shape parameter, $\mathrm{I}(x)$ denotes the indicative function illustrated as

$$\mathrm{I}(x) = \begin{cases} 1 & \text{when } [1+\zeta(\frac{x-\mu}{\sigma})] > 0 \\ 0 & \text{else} \end{cases} \tag{6}$$

when $\zeta = 0$, $\mathrm{H}(x)$ is known as the Gumbel distribution, i.e., extreme I-type; when $\zeta > 0$, $\mathrm{H}(x)$ is the Frechet distribution, i.e., extreme II-type; when $\zeta < 0$, it can be called Weibull distribution, i.e., extreme III-type. On the basis of probability distribution, i.e., $\mathrm{H}(x)$, whose inverse function can be derived easily, and then the inverse value with respect to arbitrary probability level H' can be obtained by:

$$x_{H'} = \begin{cases} \mu + \{\sigma 1 - [-\ln(H')]^{\zeta}/\zeta\}, & \zeta \neq 0 \\ \mu + \sigma\ln[-\ln(H')], & \zeta = 0 \end{cases} \tag{7}$$

According to Eq. (5), the probabilistic fitting of power spectral densities at different frequencies can be realized. Meanwhile, the parameters, i.e., μ, σ and ζ, are wholly obtained. Subsequently, with these parameters, the CP-PSDs can be mutually transformed among arbitrary cumulative probabilities.

Figs. 1 and 2 show the comparisons of CP-PSDs of track vertical profile irregularity and track alignment irregularity, which are respectively inversed from experiments. GEV distribution, and χ^2 distribution with 2 DOFs of Wuhan–Guangzhou High-speed Railway and Qinghai–Tibet Railway. From these two figures, it is clearly illustrated that there are wide variations at low cumulative probabilities if being dealt with χ^2 distribution with 2 DOFs and as opposed to this, a fairly good fitting of CP-PSDs can be realized by using GEV distribution.