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Do seismic intensity measures (IMs) measure up?

M. Grigoriu

Cornell University, Ithaca, NY14853–3501, USA

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ABSTRACT

The assumption that demand parameters D for realistic structures, i.e., complex, nonlinear dynamic systems, subjected to seismic acceleration processes $A(t)$ correlate satisfactorily with maxima $S_a(T)$ of responses of single degree of freedom (SDOF) linear systems to $A(t)$ is the cornerstone of current definitions of seismic intensity measures (IMs).

We show that, generally, $S_a(T)$ and D are weakly dependent and conclude that fragilities defined as functions of $S_a(T)$ have large uncertainties. The analysis considers linear/nonliear systems and single/multiple ordinates of $S_a(T)$. Tools of random vibration, copula models, and multivariate extreme value theory are employed to quantify the dependence between $S_a(T)$ and D.

1. Introduction

Fragilities are probabilities that structural systems enter specified damage states for given seismic intensity measures (IMs) and constitute essential tools for performance-based earthquake engineering. To be useful, IMs need to be *efficient*, i.e., structural demand parameters D conditional on IMs have small variances, and *sufficient*, i.e., the distributions of the conditional random variables $D|\text{IM}$ are completely defined for given IMs [4,11–13]. For efficient IMs, the distribution of the conditional variables $D|\text{IM}$ can be estimated from relatively small sets of structural responses. For sufficient IMs, the conditional random variables $D|$(seismic hazard) and $D|\text{IM}$ have similar distributions so that probability plots of structural damage versus IMs, i.e., fragilities, are meaningful.

IMs used currently in performance-based earthquake engineering are functionals of the seismic ground acceleration process $A(t)$, and can be divided in two groups. The first group includes functionals of samples of $A(t)$, e.g., the peak ground acceleration (PGA) and the peak ground velocity (PGV). The second group consists of functionals of filtered versions of samples of $A(t)$, e.g., single/multiple ordinates of the pseudo-acceleration response spectrum $S_a(T)$ for selected periods T. Our focus is on IMs in the second group since they are used extensively in practice.

Efficiency, sufficiency, and other properties of IMs have been studied extensively during the last two decades. Yet, these properties could not be assessed precisely since the distributions of IMs and demand parameters are not known due to the limited information on the seismic acceleration process $A(t)$. It has been proposed to (1) use concepts of the information theory to quantify the information carried by various IMs for selected demand parameter [4] and use it to rate their performance or (2) assess the performance of IMs for selected structural demand parameters based on benchmark studies [11–13]. These studies recognize that sufficient IMs may not exist and that resulting ratings of IMs may be affected by the considered information metrics and benchmark studies.

This paper examines critically the unstated assumption that responses of complex nonlinear structural systems can be predicted with satisfactory accuracy from those of linear SDOF systems. The assumption is the cornerstone of current definitions of IMs. We confine our analysis to seismic acceleration processes $A(t)$ with known probability law so that the joint distribution of $S_a(T)$ and structural demand parameters D can be found. The seismological model in [16] and other models are used to characterize the seismic acceleration process $A(t)$. Let $X_{\text{sdof}}(t)$ and $X(t)$ denote responses of linear single degree of freedom (SDOF) and complex nonlinear systems subjected to $A(t)$. These responses and, therefore, $S_a(T)$ and D, cannot be independent as functionals of the same process $A(t)$. However, they are likely to be weakly dependent for realistic structures since the stochastic processes $X_{\text{sdof}}(t)$ and $X(t)$ have very different sample properties and frequency contents. Concepts of the random vibration and the multivariate extreme value theories and copula models are used to quantify the dependence between $S_a(T)$ and D.

It is found that, for realistic structural systems, (1) the dependence between $S_a(T)$ and D is weak so that fragilities defined as functions of $S_a(T)$ have large uncertainties and (2) the fragilities defined as functions of multiple ordinates of $S_a(T)$ provide only a slight improvement over those based on single ordinates of $S_a(T)$. It is concluded that fragilities need to be defined as functions of parameters of the law of

E-mail address: mdg12@cornell.edu.

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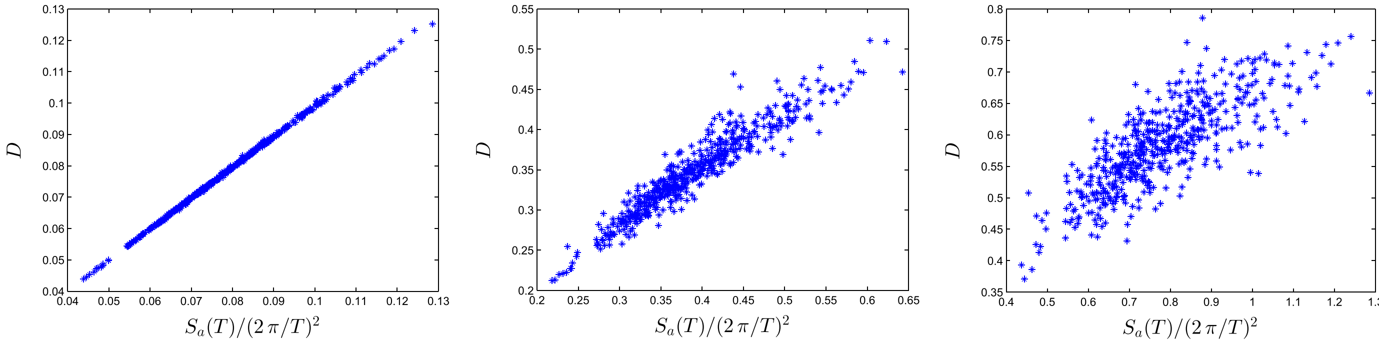

Fig. 1. Scatter plots of n=500 independent samples of $(S_a(T)/(2\pi/T)^2,\ D)$ for $\beta = 3$ and a stationary Gaussian BLWN $A(t)$ with mean 0, variance 1, and frequency band $[0,\ 10]$ scaled by 1, 5, and 10 (left, middle, and right panels).

$A(t)$, e.g., fragility surfaces of the type introduced in [10], rather than properties of functionals of samples of this process, e.g., ordinates of the pseudo-acceleration response spectrum $S_a(T)$.

2. Problem definition

Let $P_f(\xi) = P(\mathcal{D}|\text{IM} = \xi) = P(D \in I|\text{IM} = \xi)$ denote the probability that a structural system enter a damage states $\mathcal{D}$ if subjected to ground motions with scalar/vector-valued intensity measure ξ, where I defines the set of demand parameters which yield damage state $\mathcal{D}$. Fragilities are plots of $P_f(\xi)$ against ξ. Generally, the probabilities $P_f(\xi)$ are estimated from structural responses to seismic records scaled in some manner [3] so that their accuracy depends on the sample size, scaling procedure, and properties of IMs.

Suppose the seismic ground acceleration at a site can be modeled by a stochastic process $A(t)$, $t \in [0,\ \tau]$. Let $X_{\text{sdof}}(t)$ and $X(t)$ denote the response of a single degree of freedom (SDOF) linear oscillator with damping ratio ζ and period T and the response of an arbitrary structural system subjected to the same ground acceleration $A(t)$. Generally, $X(t)$ is a vector-valued process. For simplicity, we consider real-valued demand parameters of the type $D = \max_{0 \le t \le \tau} |h(X(t))|$, where τ denotes the duration of the seismic event and h maps X(t) into a real-valued response of interest, e.g., an interstory displacement or a floor acceleration. The IM of interest is the pseudo-spectral acceleration $S_a(T) = (2\pi/T)^2 \max_{0 \le t \le \tau} |X_{\text{sdof}}(t)|$. The input-output mappings $A(t) \hookrightarrow X_{\text{sdof}}(t)$; $X(t) \hookrightarrow S_a(T)$; D show that $S_a(T)$ and D are dependent random variables as functionals of $A(t)$, $0 \le t \le \tau$.

Intuition suggests that $S_a(T)$ and D are weakly dependent since they are obtained from the stochastic processes $X_{\text{sdof}}(t)$ and $X(t)$ which have very different properties as solutions of simple linear and complex nonlinear random vibration problems to $A(t)$. For example, if $A(t)$ is Gaussian, $X_{\text{sdof}}(t)$ and $X(t)$ are Gaussian and non-Gaussian processes with very different frequency bands. If this intuition is correct, fragilities defined as functions of single/multiple ordinates of $S_a(T)$ have significant uncertainties so that they are of limited practical use. The main objective of this study is to quantify the dependence between $S_a(T)$ and D and determine implicitly whether fragilities defined as functions of current IMs provide useful information for performance-based earthquake engineering.

To achieve this objective, we quantify the dependence between $S_a(T)$ and D by using a broad range of statistical tools, which are discussed in Section 3. If the dependence between $S_a(T)$ and D is very strong and weak, then $S_a(T)$ is a very good and unsatisfactory IM. Corresponding fragilities plotted against $S_a(T)$ are informative and provide at best limited information, respectively.

The remainder of this section illustrates the relationship between $S_a(T)$ and D for earthquakes of increasing intensities, outlines our formulation, and discusses briefly the computational tools used in analysis.

2.1. An illustration

Suppose $X(t)$ is the displacement of a Duffing oscillator with parameters $(\nu_0,\ \zeta,\ \beta)$ which is at rest at the initial time and is subjected to a ground acceleration process $A(t)$. Then, $X(t)$ satisfies the differential equation

$$\ddot{X}(t) + 2\,\zeta\,\nu_0\,\dot{X}(t) + \nu_0^2 (X(t) + \beta\,X(t)^3) = -A(t), \quad t \in [0,\ \tau], \tag{1}$$

with initial conditions $X(0) = 0$ and $\dot{X}(0) = 0$. If $\beta = 0$ and $\nu_0 = 2\pi/T$, then $X(t) = X_{\text{sdof}}(t)$ is the displacement of a linear oscillator with damping ratio ζ and period T. Otherwise, $X(t)$ is the response of a simple oscillator with cubic nonlinearity. The random variables $S_a(T) = (2\pi/T)^2 \max_{0 \le t \le \tau} |X_{\text{sdof}}(t)|$ and $D = \max_{0 \le t \le \tau} |X(t)|$ are dependent since they are functionals of the same input, the seismic ground acceleration process $A(t)$.

The dependence between $S_a(T)$ and D varies with the intensity of the ground motion and the magnitude of the nonlinear stiffness component. For small seismic excitation, the contribution of the cubic nonlinearity $\nu_0^2 \beta X(t)^3$ to the displacement $X(t)$ of the Duffing oscillator is insignificant so that $X(t)$ should be similar to the displacement $X_{\text{sdof}}(t)$ of the associate linear oscillator $(\beta = 0)$. In this case, the dependence between $S_a(T)$ and D is expected to be strong so that $S_a(T)$ is a very good IM. For large seismic excitations, the cubic nonlinearity $\nu_0^2 \beta X(t)^3$ contributes to $X(t)$ so that both the frequency contents and the distributions of $X(t)$ and $X_{\text{sdof}}(t)$ differ. For example, if $A(t)$ is a Gaussian process, then $X_{\text{sdof}}(t)$ and $X(t)$ are Gaussian and non-Gaussian processes. In this case, the correlation between $S_a(T)$ and D is expected to be weaker so that $S_a(T)$ is a less satisfactory IM.

These observations are consistent with the numerical results in Fig. 1 which show n=500 independent samples of the random vector $(S_a(T)/(2\pi/T)^2,\ D)$ for a Duffing oscillator with $\nu_0 = 2\pi$, $\zeta = 0.05$, and $\beta = 3$ that is subjected to a stationary Gaussian band-limited white noise (BLWN) $A(t)$ with mean 0, variance 1, and frequency band $[0,\ 10]$ during the time interval $[0,\ 20]$. The system is at rest at the initial time. The left, middle, and right panels are for ground accelerations $A(t)$ scaled by 1, 5, and 10. For small ground excitations corresponding to a scale factor of 1 (left panel), the dependence between $S_a(T)/(2\pi/T)^2$ and D is nearly perfect (the estimated correlation coefficient is almost 1). The differences between the responses $X(t)$ and $X_{\text{sdof}}(t)$ are negligible. For large ground excitations corresponding to a scale factor of 10 (right panel), the dependence between $S_a(T)/(2\pi/T)^2$ and D is weaker (the estimated correlation coefficient is 0.8107). The middle panel corresponds to moderate earthquakes, the scale factor is 5. It represents a transition between the extreme cases in the left and right panels.

The plots in Fig. 1 show, in agreement with findings in [10], that $S_a(T)$ can be viewed as a satisfactory IM for the Duffing oscillator. We attribute this performance to the fact that the Duffing oscillator is a conservative SDOF structure whose stiffness is a perturbation of the stiffness of the associated linear SDOF $(\beta = 0)$ and matches the stiffness of this system for small displacements. Yet, even in this very favorable