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Probabilistic reliability assessment of a heritage structure under horizontal loads

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ABSTRACT

This paper proposes a methodology for the probabilistic reliability assessment of heritage buildings. The procedure addresses investigation and tests on the structure and it considers the implementation of Bayesian updating techniques for a rational use of the collected information. After having described the peculiarities of ancient buildings, it is shown how probabilistic methods can be adapted to evaluate their safety. A practical application of the methodology to a relevant case study is presented, namely a historic aqueduct in Italy. The main goal is to demonstrate the effectiveness of a probabilistic approach to the reliability assessment of heritage structures.

1. Introduction

Due to environmental, economic and socio-political reasons, significance and field of application of existing structures assessments' extend rapidly, also in view of preservation of cultural heritage. Although the assessment of existing buildings and design of new buildings differs in many aspects, basic variables are handled similarly through partial factors. The partial factor method is a conservative approach that is based on fix factors and leads to a binary outcome – the reliability is verified, or not. Because of those reasons, it hardly adapts to evaluate the safety of historical buildings, where an accurate quantification of the structural reliability is essential in order to limit strengthening intervention and to predict the structural condition with increasing loads, decreasing material mechanical properties and mutated failure scenario. On the other hand, historical buildings are highly stochastic mainly because of site specific circumstances, such as the situation in which they were built and their evolution over the years. Contrary to industrial products, that are mass-produced under similar circumstances and tested repeatedly till to failure, variables here are difficult to identify and they could vary over a wide range. Furthermore, many failures occur because the scenario was not and could not have been imagined. Consequently, a probabilistic reliability assessment would lead to more rigorous quantification of the structural reliability under the assumption that all the sources of uncertainty and possible failure scenarios are properly defined. Although probabilistic methods are increasingly implemented [1–3], often resulting advantageous in comparison with those commonly used, criteria for their practical application to assess the reliability of heritage structures are still unsatisfactory.

In this paper an overall methodology for evaluating the safety of ancient buildings is proposed, that also addresses the collection of relevant information about the edifice. The procedure foresees the use of Bayesian updating techniques in order to establish probability distribution functions for the variables involved in the assessment. After having highlighted advantages and drawbacks, it is shown how a Bayesian approach can be adapted to investigate the safety of ancient structures. The reliability can then be verified implementing simple models that lead to an analytical formulation of the limit state equation. Finally the procedure has been applied to a case study, a historic aqueduct in the nearby of Pisa in Italy, affected by settlements and out of plane overturning. The survey of the aqueduct demonstrates that, when the out-of-plane rotation of the pillars is greater than 6°, the crack pattern is so significant that temporary supports must be implemented to avoid the collapse. The aim of the study is to assess how the reliability of the structure under horizontal loads, namely seismic and wind actions, changes with the inclination of the pillars, also considering the Bayesian updating of the wind speed.

1.1. Probabilistic reliability assessment

According to [4–7], a probabilistic reliability assessment is

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based on the following steps:

1. identification of relevant ultimate and serviceability limit states;
2. identification of failure modes leading to the limit state, e.g. loss of equilibrium, ultimate strength, yielding, bending, buckling, fatigue, deformation, vibration;
3. identification of basic variables that govern the failure mode, e.g. dimensions of structural elements, intensity and nature of actions, material properties, model uncertainties and internal forces;
4. definition of the probability distribution function (*pdf*) $f_i(x_i)$ for the basic variables X_i that govern the limit state;
5. definition of appropriate limit state functions expressing in the considered cases the fundamental requirement of the theory of structural reliability:

$$E < R \tag{1}$$

where the resistance R and the action effect E are suitably distributed random variables. This condition leads to the fundamental forms of the limit state functions:

$$G = R-E=0 \text{ or } Z=\frac{R}{E}=1 \tag{2}$$

where G is the so-called *safety margin* and Z the so-called *safety factor*;
6. Computation of the probability of failure:

$$p_f = P[G<0]=P[Z<1]. \tag{3}$$

The failure probability p_f in structural engineering can be obtained using a simplified approach, based on the estimation of the reliability index β, which is a function of p_f,

$$\beta = -\Phi^{-1}(p_f); \tag{4}$$

where Φ is the standard normal cumulative distribution function (CDF);
7. verification of the structural reliability: the goal for reliability analysis is to document the achievement of the target reliability, reflecting the 'accepted' level of risk in terms of possible failure consequences in a given reference time period. The following verification formats are considered:

$$\left(1-p_f\right) > \left(1 - p_d\right) \text{ or, equivalently, } \beta > \beta_t \tag{5}$$

where the target reliability is represented by $(1-p_d)$ or β_t.

A basic step of the probabilistic approach is represented by the definition of appropriate *pdf*s for the random variables X_i. Depending on the available information, probability models can be built following different approaches: if some natural variability of a parameter is evidenced through a set of measured values, the classical approach consists in using statistical inference methods; however, this is not the usual case in the assessment of existing structures, where obviously only a limited number of tests can be performed on the actual structure. Nonetheless several information regarding the structural features may be available from other sources, such as original drawings and calculation, Codes and Standards in force when the structure was designed, other buildings presenting similar characteristics, a study of the available literature, information from theoretical models, expert's judgments based on experience, and modern databases of test results. Since the nature of these indirect information is often qualitative, the Bayesian approach is the more suitable way for defining probabilistic models for the assessment of existing buildings.

1.2. *The Bayesian analysis in the reliability assessment of existing structures*

A Bayesian approach is based on 2 concepts: (i) the interpretation of probability as the degree of belief attributed to a certain event, given the current state of information; (ii) the application of the Bayes Theorem for updating a prior belief when more evidence becomes available. Furthermore, it provides a logical basis for the estimation of the parameters of an underlying probability model: while in the classical approach the uncertainty in the parameter estimation is expressed through confidence interval, in a Bayesian setting the unknown parameter θ is assumed as a random variable.

In the case of parameter estimation, we may often have some knowledge or intuitive judgments of the possible values of the parameter; as additional information becomes available (such as the results of a series of tests or experiments), the prior assumption on the parameter may be modified formally through the Bayes Theorem as shown below [8]:

$$f''(\theta \mid \epsilon) = \frac{P(\epsilon\theta)f'(\theta)}{\int_{-\infty}^{+\infty} P(\epsilon\theta)f'(\theta)d\theta}, \tag{6}$$

The term $P(\varepsilon|\theta)$ is the conditional probability, or likelihood, of observing the experimental outcome ε assuming that the value of the parameter is θ. The denominator is independent of θ; this is simply a normalizing constant required to insure that $f''(\theta|\varepsilon)$ is a proper *pdf*. In this way, all sources of uncertainty associated with the estimation of the parameter can be combined formally through the total probability theorem; moreover, subjective judgments based on intuition, experience, or indirect information are incorporated systematically with observed data (through the Bayes Theorem) to obtain a balanced estimation.

The expected value of θ is commonly used as the point estimator of the parameter. Hence, the updated estimate of the parameter θ, in the light of the observational data ε, is given by:

$$\hat{\theta}'' = \mathbb{E}[\Theta \mid \epsilon] = \int_{-\infty}^{\infty} \theta f''(\theta \mid \epsilon)d\theta \tag{7}$$

where Θ is the random variable whose values represent possible values of θ. The uncertainty in the estimation of the parameter can be included in the calculation of the probability associated with a value of the underlying random variable. For example, if X_i is a random variable whose probability distribution has a parameter θ, its probability is:

$$p(X_i \le a) = \int_{-\infty}^{\infty} p(X_i \le a \mid \theta) f''(\theta \mid \epsilon)d\theta. \tag{8}$$

2. Challenges in applying probabilistic methods to historic buildings

2.1. *General considerations*

With historic buildings it is meant not only important monuments, but also vernacular heritage mainly made of masonry and wood. Applying probabilistic methods to heritage structures can be particularly difficult, for the following reasons:

1. They have been often designed according to empirical bases or architectural canons, instead of formal design approach and recognized theories or normative prescription;
2. The acquisition of data is a complicated process [9], since the probabilistic description of material properties is affected by great uncertainties, originating for example from the