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Reducing forecast uncertainty by using observations in geotechnical engineering

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ABSTRACT

Especially in geotechnical engineering, a high level of uncertainty in the design of structures is present. Standards and guidelines recommend the observational method for projects with a high level of uncertainty and when the geotechnical behaviour is difficult to predict. The behaviour of a complex geotechnical problem is measured in each constructions step and these measurements are compared to simulation results. As a next step one compares the used model parameters and assumptions. In case of big differences, one adapts the system in order to improve it for the next construction step. However, this design approach is based on engineering judgement in combination of deterministic approaches, which are adapted sequentially whenever new observations are available. This formulates the need for a sound mathematical and statistical framework, which allows to combine measurement to quantify forecast uncertainty.

This paper explains two mathematical concepts for data assimilation. The sequential and variational data assimilation consider a stochastic system, which is updated by uncertainty observations. These concepts reduce the simulation uncertainty by using observations. Two case studies show the applications of both concepts in geotechnical engineering problems. The first case study is discussing the possibilities and limitations of the sequential data assimilation concept in a theoretical example, whereas the second case study is demonstrating the combination of settlement measurements and a stochastic subsoil model by means of variational data assimilation. Additionally, the concept of forecast uncertainty quantification is demonstrated in the second case study. At the end a brief review of the data assumption concepts and the given forecast uncertainty quantification approach is given together with conclusions for further research.

1. Introduction

Design and construction of geotechnical structures often involve a wide range of uncertainties, which are generally associated with the interpretations and assessment of geotechnical properties from a set of data. Eurocode 7 [2] recommends to use the observational method (OM) in projects with a high level of uncertainty and when the geotechnical behaviour is difficult to predict. This design approach enables the engineer to address the uncertainties by continuously predicting, observing and altering the design during construction [17]. Since Peck [16] first formulated the design philosophy of the OM, this design methodology has been contentiously refined and improved. The major principles of the OM are to initially assess acceptable limits of behaviour and the range of possible behaviour for a structure, subsequently followed by monitoring and evaluation of the actual behaviour. During the design phase, it is of great importance that prospective deviations in geotechnical behaviour and associated failure mechanisms are identified so that appropriate actions can be devised in an early stage of the design process. In case of deviation from initial assessments and established limits during construction, actions have to be preformed according to the plan elaborated in the design phase.

Up to now, the OM is based purely on expert judgement and on a deterministic design approach [7,13,16,17,19] amongst others; recently, some researchers try to incorporate the uncertainty of the soil properties using standard statistical approaches in a Bayesian framework as given in [21]. However, the variability of the simulation model is not considered explicitly and the uncertainties of the observations are not taken into account because they are considered as small in comparison to the subsoil uncertainty.

In this contribution, I compare two statistical frameworks, which can consider both uncertainties of the subsoil and of the observations. Moreover, I show that one can also incorporate the observations in the probabilistic analysis of a complex geotechnical problem. Additionally, I show how to quantify the forecast

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uncertainty based on methods from numerical weather prediction. The possibilities and limitations of these approaches are discussed in case studies, which shall help the reader to grasp the basic ideas more clearly. This paper ends with a summary and review of the presented data assimilation approaches and forecast uncertainty quantification as well as future research steps.

2. Combination of uncertain observations and probabilistic forecasts

Apart from geotechnical engineering, uncertainties in model parameters are a dominant source of uncertainty for e.g. hydrological models and models in numerical weather prediction. These engineering models are approximating complex system, e.g. non-linear soil behaviour or soil–structure interaction problems, and these models are therefore not perfect. In contrast, measurements like deformations are describing the behaviour of an engineering system more precisely due to their smaller scatter.

The basic idea of data assimilation is to incorporate the information of both the uncertain simulation model and the imprecise measurements. Fig. 1 shows the behaviour (or state) of a system, which is unknown. By using a simulation model for approximation, the unknown system behaviour, one can derive the simulation uncertainty using probabilistic approaches. By means of data assimilation techniques one can calculate the combined uncertainty by combining the simulation uncertainty and observation uncertainty. As given in Fig. 1 qualitatively, the combined uncertainty is smaller than the simulation uncertainty and bigger than the observation uncertainty. One can employ sequential and variational data assimilation approaches as described by Evensen [6] amongst others.

2.1. Sequential data assimilation

One of the most used approaches in sequential data assimilation is the Ensemble Kalman Filter (EnKF). It was originally proposed as a stochastic or Monte Carlo alternative to the Kalman Filter (EKF) by Evensen [6]. The EnKF has gained popularity because of its simple conceptual formulation and relative ease of implementation.

The basic idea of the Ensemble Kalman Filter (EnKF) is the Kalman Filter. The Kalman Filter is an efficient framework to update the modelling system $\mathbf{x}^f$, which is also called forecast system, by observations $\mathbf{y}$. This linear update of the state of the system $\mathbf{x}$ is given in Eq. (1). Herein, the state of the system $\mathbf{x}$ is a vector consisting of all variables, which are describing the conditions of the system (like model parameters and/or deformations). The posterior estimate of the state of the system $\mathbf{x}^a$ is given by

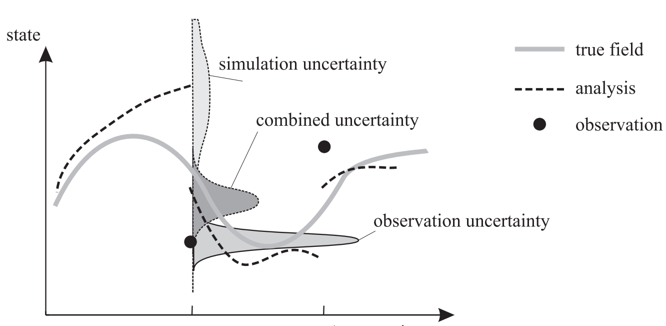

Fig. 1. Concept of data assimilation with the uncertainty of the simulation, the observation and the combined uncertainty for a given state.

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}\left(\mathbf{y} - \mathbf{H}\mathbf{x}^f\right) \quad (1)$$

and the analysis error covariance $\mathbf{P}^a$ is given by

$$\mathbf{P}^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^f \quad (2)$$

where $\mathbf{x}^f$ is the prior estimate with n state variables, $\mathbf{y}$ is the observation vector of m observations, $\mathbf{H}$ is the observation operator matrix (dimension $m \times n$) which maps state variables to observations like deformation measurements, $\mathbf{I}$ is an identity matrix (dimension $n \times n$), and $\mathbf{P}^f$ is the forecast background error covariance (dimension $n \times n$). The Kalman gain matrix $\mathbf{K}$ (dimension $n \times m$) is defined as

$$\mathbf{K} = \mathbf{P}^f\mathbf{H}^T\left(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R}\right)^{-1} \quad (3)$$

where $\mathbf{R}$ is the observation error covariance (dimension $m \times m$). $\mathbf{P}^f\mathbf{H}^T$ is the predicted covariance between the states and observed variables, and $\mathbf{H}\mathbf{P}^f\mathbf{H}^T$ is the predicted error covariance of the observed variables; this predicted error covariance is quantifying the forecast uncertainty of the system, which is represented by the state vector $\mathbf{x}$.

Note that the Kalman Filter is only correct for linear systems. This drawback can be overcome through the Monte-Carlo approach within the Ensemble Kalman Filter. Herein, a sufficient large ensemble consisting of random samples is used to represent the system behaviour, which allows the modelling of non-linear systems in various fields such as atmospheric or oceanic sciences [6]. This implies that the vector and matrices of the equations above become large, which states a computational challenge for large and complex systems. These computational requirements are affordable and comparable to other popular sophisticated assimilation methods such as the variational data assimilation method, as stated by Evensen [6].

2.2. Variational data assimilation

The basic idea of variational data assimilation is to find the initial conditions of a model, such as to minimize some scalar quantity J. The cost function $J[\mathbf{x}]$ is a function of the state vector $\mathbf{x}$, which is a vector of all variables describing the conditions of the system like model parameters and/or deformations. The cost function defines a global measure of the simultaneous misfit between $\mathbf{x}$, the current guess of the model state, and to the observations $\mathbf{y}$. The cost function J simultaneously penalizes a bad fit between the model state $\mathbf{x}$ and the background, and the model state and the predicted observations, given in the following equation:

$$J[\mathbf{x}] = \tfrac{1}{2}\left(\mathbf{x}^f - \mathbf{x}\right)^{\mathrm{T}}\mathbf{B}^{-1}\left(\mathbf{x}^f - \mathbf{x}\right) + (\mathbf{y} - \mathbf{H}\,\mathbf{x})^{\mathrm{T}}\,\mathbf{R}^{-1}(\mathbf{y} - \mathbf{H}\,\mathbf{x}) \quad (4)$$

where $\mathbf{B}$ is the background error covariance matrix, $\mathbf{y}$ is the set of observations made at time t and $\mathbf{R}$ is the observational error covariance matrix. The vector $\mathbf{y} - \mathbf{H}\mathbf{x}$ is the residual. The covariance matrix $\mathbf{P}^a$ of the resulting state vector $\mathbf{x}^a$ is equal to the inverse of the Hessian matrix of the objective function evaluated at $\mathbf{x}^a$.

Using variational data assimilation, one assumes that the errors are unbiased and standard normally distributed. Additionally, the model is assumed to represent the system behaviour perfectly. Amongst others, Evensen [6] states that 3D-VAR stands for the variational data assimilation, in which no account is taken of the time that observations are taken. The three dimensions are spatial. 4D-VAR stands for variational data assimilation within three space dimensions plus one time dimension.

Evensen [6] reports that the drawback of the variational

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