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On reliability of systems with moving material subjected to fracture and instability

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ABSTRACT

The reliability of systems with moving cracked elastic and isotropic material is considered. The material is modeled as a moving plate which continually has a crack on the edge. The plate is subjected to homogeneous tension acting in the traveling direction and the tension varies temporally around a constant value, the set tension. The tension and the length of the crack are modeled by an Ornstein–Uhlenbeck process and an exponential Ornstein–Uhlenbeck process, respectively. Failure is regarded as the state at which the plate becomes unstable or fractures (or both) and a lower bound for the reliability of the system is derived. Considering reliability of the system leads to first passage time problems and, in solving them, a known explicit result for the first passage time of an Ornstein–Uhlenbeck process to a constant boundary is exploited. A change in the set tension has opposite effects on the probabilities of instability and fracture, and a safe range of set tension is studied. Numerical examples are computed for material and machine parameters typical of paper and printing presses. The results suggest that tension variations may significantly affect the pressroom runnability.

1. Introduction

Models of axially moving continua are commonly used to describe mechanical systems in industry, such as band saws, transmission chains or moving paper webs in printing presses. Thus, a large amount of research have been dedicated to the mechanics of axially moving media, with different mechanical and material models. The most common models of axially moving materials are traveling flexible strings, membranes, beams and plates. The first paper on axially moving materials dates back to 1897, when Skutch [1] published a paper concerning the elastic string model. Studies of moving elastic strings were continued by Sack [2] and Archibald and Emslie [3], who were also the first published authors on axially moving materials in English. Later studies of axially moving materials concern, e.g., stability of traveling two-dimensional elastic [4,5], orthotropic [6,7] and viscoelastic [8,9] plates. Extensive literature reviews on the studies of axially moving materials can be found in [5,7,9].

Although random variations may be significant when the performance of the system is considered, only a few studies of axially moving materials with a stochastic setup can be found in the literature. In studies by Tirronen et al. [10,11] and Banichuk et al. [12], an axially moving elastic and isotropic cracked plate was studied in the case in which parameter values include uncertainty. The problem parameters were modeled by random variables, and critical regimes were obtained for velocity and tension, to which the plate was assumed to be subjected. However, in [10,12,11] longevity of the system was not considered, although it is of interest from the practical point of view. Temporal variations were not addressed in [10,12,11] although, e.g., in a printing press tension is known to vary with respect to time [13].

The present paper extends [10–12] by modeling randomly varying problem parameters as stochastic processes, which provide natural models for temporal stochastic variations of the system and enable examination of its longevity. In this paper, the tension variations are described by a stationary Ornstein–Uhlenbeck process. With this model, the tension has a constant mean value, the set tension, around which its value fluctuates temporally. The Ornstein–Uhlenbeck process can be considered as the continuous-time analogue of the discrete-time AR(1) process, the simplest model for a time series in which data points have dependency. It provides a mathematically well-defined continuous-time model for fluctuations of systems whose measurements contain white noise [14, Chapter 4]. Moreover, a stationary process describes random fluctuations of a system which has settled down to a steady state and whose statistical properties do not depend on when they are measured [14, Sections 3.7]. In a printing press, the draw variations contain specific high/low frequency components as well as white noises [13]. In addition, tension surges are likely to occur during start-up and shutdown operations [15]. The stationary Ornstein–Uhlenbeck process can be regarded as a

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simplified model of tension variations in a pressroom.

Another stochastic quantity in the considered model is related to fracture. The plate is assumed to continually have a weak spot caused by a defect from which a fracture begins to propagate if the tension value is too high. Defects in a paper web can be classified into two categories: microscopic defects, which originate from the natural disorder of paper such as variations of fibre orientation and density, and macroscopic defects introduced during the papermaking and transportation processes [16]. Macroscopic defects include, e.g., edge cracks. Such cracks may occur as a consequence of insufficient roll edge protection during handling and storage [17]. A cut or nick in the edge of the roll may cause multiple edge cracks in the sheet in a localized area. Edge cracks may also occur randomly in the sheet and be caused by stress formed from running a high roll edge through a nip [17].

In this study, defects of the material are modelled as cracks. It is assumed that the lengths of the cracks are typically small compared to the width of the plate. As sharp edge cracks perpendicular to the travelling direction are most critical in terms of fracture, such cracks are considered. Moreover, the crack lengths are modeled by a stationary exponential Ornstein–Uhlenbeck process which is assumed to be independent of the process describing tension. With the exponential Ornstein–Uhlenbeck process, the length of a single crack obeys lognormal distribution.

In this study, reliability of the system is of interest. Failure is considered as a state at which the plate becomes unstable or fractures (or does both). For instability, the results obtained in [5] are utilized. To study fracture of the plate, linear elastic fracture mechanics (LEFM) is applied. The probabilities that the plate remains stable and does not fracture are considered separately, and a lower bound for the reliability of the system is obtained.

Considering the probability of stability leads to a first passage time problem. In solving it, the spectral expansion of the first passage time distribution of an Ornstein–Uhlenbeck process to a constant boundary given in [18] is exploited. By using the analytical expression for the first passage time distribution one avoids discretization error that would result from plain Monte Carlo simulation.

The probability that the plate does not fracture is a solution to a first crossing time problem of two stochastic processes. This probability may be approximated from below by the probability that the tension does not hit the minimum critical tension obtained from the process describing the length of the crack. Thus, the result for the first passage time to a constant boundary may also be exploited in obtaining a lower bound for the probability that the plate does not fracture.

The model is studied for material and machine parameters typical of paper and printing presses. The effect of the parameters of the stochastic quantities on the reliability of the system is investigated numerically. As a change in the set tension has opposite effects on the probabilities of instability and fracture, a safe range of set tension is also studied.

A web break is an important runnability issue in pressrooms [19] and reducing the number of web breaks is a major concern. Accordingly, the web break is a widely investigated subject in the pulp and paper industry [13]. However, web breaks are rare events, and experimental studies require data from a large number of rolls to determine the causes of web breaks [19]. Thus, mathematical modeling may provide an efficient tool to study system performance.

This study aims at developing mathematical models and techniques for estimating the reliability of systems with moving cracked material. Combined with data of defects and tension, the models developed in this study can be used for predicting the reliability of systems with moving material in terms of fracture and instability. For printing processes, such data can be obtained by automated inspection systems developed for quality control [20] and devices designed for tension profile measuring [21].

2. Problem setup

Considered here is a system in which a cracked elastic and isotropic band travels, at some state, unsupportedly from one roller (support) to another and is subjected to tension acting in the traveling direction. In this section, a mathematical model for the moving band is presented. The model described below is similar to the deterministic one presented in [5].

To study the behavior of the band, consider a rectangular part of it that occurs between the supports

$$\mathcal{D} = \{(x, y):\ 0 < x < \ell,\ -b < y < b\} \tag{1}$$

in x, y coordinates (see Fig. 1). It is assumed that between the supports the band travels in the x direction with constant velocity $V_0 > 0$. Above, ℓ is the length of the span between the supports and b is half of the band width.

The part $\mathcal{D}$ is modeled as an elastic and isotropic plate having constant thickness h and mass m (per unit area of the middle surface of the plate). The sides of the plate

$$\{x = 0,\ -b < y < b\} \quad \text{and} \quad \{x = \ell,\ -b < y < b\} \tag{2}$$

are assumed to be simply supported, and the sides

$$\{y = -b,\ 0 < x < \ell\} \quad \text{and} \quad \{y = b,\ 0 < x < \ell\} \tag{3}$$

are free of tractions. Moreover, the plate has a constant Poisson ratio ν and Young modulus E.

We study the probability that a band of given length travels through the system of rollers without failure. In this, it is assumed that the material continues and remains similar after the considered band.

2.1. Tension

The plate element (1) is subjected to homogeneous tension acting in the x direction. Temporal fluctuations of tension are modelled by a continuous-time stochastic process

$$T = \{T(s),\ s \geq 0\} \tag{4}$$

in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. In (2), s denotes the length of the part that has travelled through the open draw, see Fig. 1.

Furthermore, we describe the tension by a stationary Gaussian Markov process. For definitions of stationarity and the Markov property, see [14, Sections 3.7 and 3.2], respectively. By a stationary process one describes the random fluctuations of a system which has settled down to a steady state and whose statistical properties do not depend on when they are measured [14, Sections

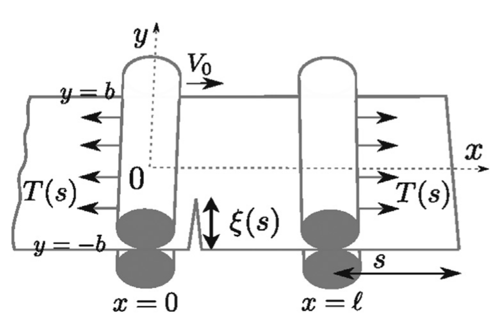

Fig. 1. The part of the band that is traveling between the supports is modeled as a moving plate that is tensioned at the supported edges by homogeneous tension $T(s)$. The drawing is adapted from Fig. 1 in [10].