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The parameterization of all stabilizing modified Smith predictors for non-square time-delay plants

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Abstract: The modified Smith predictor is well known as an effective time-delay compensator for a plant with large time-delays, and several papers on the modified Smith predictor have been published. The parameterization of all stabilizing modified Smith predictors for multiple-input/multiple-output time-delay plants was obtained by Yamada et al. However, they do not examine the parameterization of all stabilizing modified Smith predictors for non-square time-delay plants. The purpose of this paper is to expand the result by Yamada et al. and to propose the parameterization of all stabilizing modified Smith predictors for non-square time-delay plants. Control characteristics of the control system using obtained parameterization of all stabilizing modified Smith predictors are also given.

Keywords: Multiple-Input/Multiple-Output Plant; Time-Delay System; Smith Predictor; Parameterization; Non-Square System.

1. INTRODUCTION

In this paper, we examine the parameterization of all stabilizing modified Smith predictors for non-square time-delay plants. The Smith predictor was proposed by Smith to overcome time-delays (Smith, 1959), it is well known as an effective time-delay compensator for a stable plant with large time-delays (Smith, 1959; Sawano, 1962; Hang and Wong, 1979; Watanabe and Ito, 1981; Watanabe and Sato, 1984; De Paor, 1985; Deshpande and Ash, 1988; De Paor and Egan, 1989; Astrom et al., 1994; Matausek and Micic, 1996; Watanabe, 1997; Kwak et al., 1999). The Smith predictor in Smith (1959) cannot be used for plants having an integral mode, because a step disturbance will result in a steady state error (Sawano, 1962; Hang and Wong, 1979; Watanabe and Ito, 1981). To overcome this problem, Watanabe and Ito (1981), Astrom et al. (1994) and Matausek and Micic (1996) proposed a design method for modified Smith predictor for time-delay plants with an integrator. Watanabe and Sato expanded the result in Watanabe and Ito (1981) and proposed a design method for modified Smith predictors for multivariable systems with multiple time-delays in inputs and outputs (Watanabe and Sato, 1984).

Because the modified Smith predictor cannot be used for unstable plants (Sawano, 1962; Hang and Wong, 1979; Watanabe and Ito, 1981; Watanabe and Sato, 1984; De Paor, 1985; Deshpande and Ash, 1988; De Paor and Egan, 1989; Astrom et al., 1994; Matausek and Micic, 1996; Watanabe, 1997), De Paor (1985), De Paor and Egan (1989) and Kwak et al. (1999) proposed a design method for modified Smith predictors for unstable plants. Thus, several design methods of modified Smith predictors have been published.

On the other hand, another important control problem is the parameterization problem, the problem of finding all stabilizing controllers for a plant (Zames, 1981; Youla et al., 1976; Desoer et al., 1980; Vidyasagar, 1985; Morari and Zafiriou, 1989; Glaria and Goodwin, 1994; Yamada, 2001; Nobuyama and Kitamori, 1990, 1991). The parameterization of all stabilizing controllers for time-delay plants was considered in Nobuyama and Kitamori (1990, 1991), and that of all stabilizing two-degree-of-freedom controllers for time-delay plants was considered in Mirkin and Zhong (2003), which enable to analyze the set-point and disturbance responses separately without the need to compromise them. In addtion, Zhang et al. (2006) proposed a new parameterization, which does not depend on the coprime factorization and has similar form to that of the Youla parameterization for stable plants (Youla et al., 1976). However, the parameterization of all stabilizing modified Smith predictors has not been obtained. Yamada et al. gave the parameterization of all stabilizing modified Smith predictors for minimum-phase single-input/single-output time-delay plants (Yamada and Matsushima, 2005) and for non-minimum-phase single-input/single-output time-delay plants (Yamada et al., submitted). In addition, the parameterization of all stabilizing modified Smith predictors for multiple-input/multiple-output time-delay plants was obtained by Yamada et al. (2010). Since the parameterization of all stabilizing modified Smith predictors was obtained, we could express previous studies of modified Smith predictors in a uniform manner and could design modified Smith predictors systematically.

However, the proposed parameterization in Yamada et al. (2010) examines for only multiple-input/multiple-output time-delay plants, whose number of the input is equal to that of the output. That is, their parameterization can only be used for square time-delay plants. In case of controlling the time-delay plants in practice, it is not always that the time-delay plant is square. Therefore, it is necessary to consider for non-square time-delay plants, but no paper examines.

The purpose of this paper is to propose the parameterization of all stabilizing modified Smith predictors for non-square time-delay plants. First, the structure and necessary characteristics of modified Smith predictors described in past studies in Smith (1959); Sawano (1962); Hang and Wong (1979); Watanabe and Ito (1981); Watanabe and Sato (1984); De Paor (1985); Deshpande and Ash (1988); De Paor and Egan (1989); Astrom et al. (1994); Matausek and Micic (1996); Watanabe (1997); Kwak et al. (1999) are defined. Next, the parameterization of all stabilizing modified Smith predictors for non-square time-delay plants is proposed, for both stable and unstable plants. Finally, control characteristics of the control systems using this parameterization are also given.

Notation

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of stable proper real rational functions.
$\Re\{\cdot\}$	The real part of $\{\cdot\}$.
A^T	Transposed matrix of A.

2. MODIFIED SMITH PREDICTOR AND PROBLEM FORMULATION

Consider the control system:

$$\begin{cases} y(s) = G(s)e^{-sT}u(s) + d(s) \\ u(s) = C(s)\left(r(s) - y(s)\right) \end{cases}, \tag{1}$$

where $G(s)e^{-sT}$ is the multiple-input/multiple-output time-delay plant, $G(s) \in R^{m\times p}(s)$, $T > 0$ is the time-delay, $C(s)$ is the controller, $y(s) \in R^m(s)$ is the output, $u(s) \in R^p(s)$ is the control input, $d(s) \in R^m(s)$ is the disturbance and $r(s) \in R^m(s)$ is the reference input. It is assumed that $G(s)$ is stabilizable and detectable and the number of the output is smaller than or equal to that of the input, that is $m \le p$.

According to Sawano (1962); Hang and Wong (1979); Watanabe and Ito (1981); Watanabe and Sato (1984); De Paor (1985); Deshpande and Ash (1988); De Paor and Egan (1989); Astrom et al. (1994); Matausek and Micic (1996); Watanabe (1997); Kwak et al. (1999), the modified Smith predictor $C(s)$ is decided by the form:

$$C(s) = C_1(s)\left(I + C_2(s)e^{-sT}\right)^{-1}, \tag{2}$$

where $C_1(s) \in R^{p\times m}(s)$ and $C_2(s) \in R^{m\times m}(s)$. In addition, using the modified Smith predictor in Sawano (1962); Hang and Wong (1979); Watanabe and Ito (1981); Watanabe and Sato (1984); De Paor (1985); Deshpande and Ash (1988); De Paor and Egan (1989); Astrom et al. (1994); Matausek and Micic (1996); Watanabe (1997); Kwak et al. (1999), the transfer function from $r(s)$ to $y(s)$ of the control system in (1), written as

$$y(s) = \left(I + G(s)C(s)e^{-sT}\right)^{-1} G(s)C(s)e^{-sT}r(s), \tag{3}$$

has finite numbers of poles. That is, the transfer function from $r(s)$ to $y(s)$ of the control system in (1) is written as

$$y(s) = \bar{G}(s)e^{-sT}r(s), \tag{4}$$

where $\bar{G}(s) \in RH_\infty^{m\times m}$. Therefore, we call $C(s)$ the modified Smith predictor if $C(s)$ takes the form of (2) and the transfer function from $r(s)$ to $y(s)$ of the control system in (1) has finite numbers of poles.

The problem considered in this paper is to obtain the parameterization of all stabilizing modified Smith predictors $C(s)$ for non-square time-delay plants.

3. THE PARAMETERIZATION OF ALL STABILIZING MODIFIED SMITH PREDICTORS FOR STABLE PLANTS

In this section, we propose the parameterization of all stabilizing modified Smith predictors $C(s)$ for stable plants $G(s)e^{-sT}$.

This parameterization is summarized in the following theorem.

Theorem 1. $G(s)e^{-sT}$ is assumed to be stable. The parameterization of all stabilizing modified Smith predictors $C(s)$ takes the form:

$$C(s) = Q(s)\left(I - G(s)Q(s)e^{-sT}\right)^{-1}, \tag{5}$$

where $Q(s) \in RH_\infty^{p\times m}$ is any function.

Proof. First, the necessity is shown. That is, we show that if $C(s)$ in (2) makes the control system in (1) stable and makes the transfer function from $r(s)$ to $y(s)$ of the control system in (1) have finite numbers of poles, then $C(s)$ takes the form of (5). From the assumption that $C(s)$ in (2) makes the transfer function from $r(s)$ to $y(s)$ of the control system in (1) have finite numbers of poles,

$$\left(I + G(s)C(s)e^{-sT}\right)^{-1} G(s)C(s)e^{-sT}$$