Problem for Light Pipes and a Solution Proposal

G. Lizárraga-Lizárraga*[1], T. E. Salais-Fierro[2], J. A. Saucedo-Martínez[3], R. Garza-Morales[4] and J. Sánchez-Aguilar[5]

[1,2,3,4] Facultad de Ingeniería Mecánica y Eléctrica
Universidad Autónoma de Nuevo León
Monterrey, México
*giovanni@cimat.mx
[5] Ciencias Básicas
Instituto Tecnológico de Querétaro
Querétaro, México

ABSTRACT
This work deals with the light pipe problem. In this problem it is necessary to both put circles inside circles and circles inside a rectangle. A mathematical model that considers the thickness of the pipes is introduced. This generates a new optimization problem that is harder to solve than other packing problems. The mathematical description of the problem is introduced and some ideas of how to solve the problem are explained.

Keywords: light pipes, optimization, heuristics, transportation.

1. Introduction

The distribution of products from the factory to the consumers is a very important issue in logistics [1]. In fact, the transportation of a product represents 40% of its cost [2]. This means that a more efficient packing and route tracing have a great impact in the competitiveness of a company.

The light pipes distribution problem has some special characteristics that make it different from classics distribution problems. For example, the weight of the pipes does not represent a constraint. Therefore, many pipes can be stacked one over the other without damaging the pipes at the bottom. Another characteristic of this problem is that the pipes can be introduced one inside the other when the internal radius of one of them is greater than the external radius of the other one. This can be done several times, in such a way that it can be a pipe inside another pipe that, at the same time, is inside a greater pipe and so on.

Another important point to consider is that the truck carrying the pipes may be loaded with pipes of several different clients from different locations. Then, whether the pipes for the first client are at the bottom of the truck, an additional cost for relocations of pipes must be considered. Furthermore, an additional way to reduce the cost of transportation is to load the truck in such a way that the reload is minimized.

In this work, the problem of maximizing the load of pipes in a truck is analyzed, disregarding other aspects of the problem that will be treated in a future work.

The main goals of this paper are to introduce a mathematical model for this problem, and to propose some potential ways to solve it. In the next Section, a mathematical formulation of the problem is presented. In Section 3, a description of Evolutionary Strategy, a powerful optimization algorithm, is explained. In Section 4, the result of the experiments are reported. Finally, in Section 5 the conclusion and some ideas for future work are presented.

2. Materials and methods

2.1 The packing problem

Packing a set of light pipes is related to several other problems in literature, for example the Circle Packing in a Square (CPS) [3]. In CPS, there is a set of n circles whose centers have coordinates (x_i, y_i), for $i = 1, 2, 3, \ldots, n$. All circles have the same radius r_n, but this radius is unknown *a priori*. The centers of the circles are located inside a square whose sides have a length S. The solution of the problem consists of locating the centers of the circles in such a way that the radius r_n is maximal, the circles do not overlap and all circles are contained inside the square of side S.

Another similar problem is packing Unequal Discs in a Circle (UDC) [4]. In this problem, there are n disks of different fixed radius r_i and centers with coordinates (x_i, y_i), for $i = 1, 2, 3, \ldots, n$. The solution of the problem consists of locating the center of the circles in such a way that the radius r_n of a circle that contains all disks is minimized.

In this work, we study the case where the transversal area of the pipes is used in the optimization, ignoring the length of the pipes. The objective is to maximize the sum of the transversal areas of the pipes that are loaded in the truck. The rear view of the box of the truck is rectangular. So, the light pipes packing problem (LPP) has characteristics of both CPS and UDC. For example, it is necessary to put several circles inside a rectangular area, and at the same time to put circles inside other circles.

The LPP has differences with respect the other problems. For example, in the LPP the radii of the circles are fixed from the start and cannot be modified like in CPS. Another difference is that in the LPP, when circles are put inside a circle, the radius of the containing circle is fixed, unlike the UDC. Moreover, in the LPP it is needed to carry the set of pipes is as small as possible, but in this work the focus is in the problem possible that not all the pipes can be loaded in the truck, and must be loaded in another truck. The ideal result is that in which the number of trucks of loading a single truck with as many pipes as possible. Figure 1 shows an example of a truck loaded with a light pipes. The rectangle represents the rear view of box of the truck. The circles represent the transversal view of the pipes loaded in the truck.

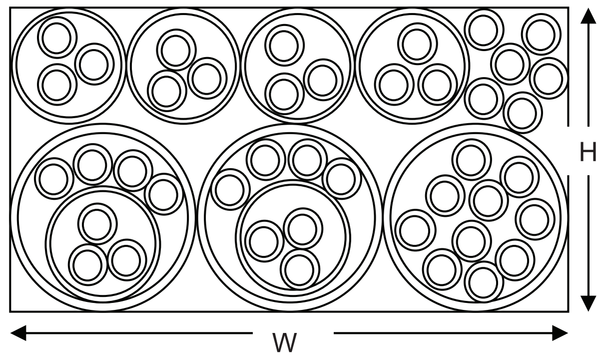

Figure 1. An example of a truck loaded in the LPP problem.

2.2 Mathematical model for LPP

In this subsection, a mathematical model for the LPP problem is presented. There is some terminology that will be used for the rest of the paper. First, the transversal area of a pipe is a ring, so the terms ring and transversal area of a pipe are used as equivalents, and the demonstrations and definitions given for one of them is valid for the other one.

A ring is a geometric figure formed by two concentric circles of different diameter. In this paper, a ring is represented with the letter P. For a set of n rings the *i-th* element is called P_i. The radius of the greater circle of P_i is denoted by R_i and the radius of the smaller circle of P_i is denoted by r_i (see Figure 2). The greater circle of P_i is denoted by PR_i and the smaller circle of P_i is denoted by Pr_i.

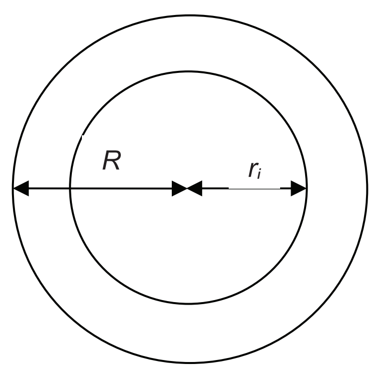

Figure 2. Radius of a ring.