Toward Optimal Control of Flat Plate Photobioreactors: the Greenhouse Analogy?

G. van Straten*, P.M. Slegers*, L.G. van Willigenburg*, R. Bosma, A.J.B. van Boxtel*, R.H. Wijffels****

**Wageningen University, Systems and Control, 6708 WG Wageningen, The Netherlands (Tel.+31 317 483331; e-mail: gerrit.vanstraten@wur.nl)*
*** Wageningen University, Bioprocess Engineering,, 6703 HD Wageningen, The Netherlands*

Abstract: The cultivation of algae in photo-bioreactors shows similarities to crop cultivation in greenhouses, especially when the reactors are driven by sun light. Advanced methodologies for dynamic optimization and optimal control for greenhouses are known from earlier research. The aim here is to extend these methodologies to microalgae cultivated in a flat plate photo-bioreactor. A one-state space model for the algal biomass in the reactor is presented. The growth rate vs. light curve is parameterized on the basis of experimental evidence. Spatial distribution of light and growth rate between the plates is also considered. The control variable is the dilution rate. Dynamic optimal control trajectories are presented for various choices of goal function and external solar irradiation trajectories over a horizon of 3 days. It was found that the algae present in the reactor at final time represent a value for the future. Numerical and theoretical results suggest that the control is bang-(singular-)bang, with a strong dependence on the weather. The optimal biomass also depends on the available light, and achieving it to reach a new optimal steady cycle after a prolonged change in weather may take several days. A preliminary theoretical analysis suggests a control law that maximizes the effective growth rate. The analysis shows that like in the greenhouse case, the co-state of the algal biomass plays a pivot role in developing on-line controllers.

Keywords: Optimal control; Photo-bioreactor; Microalgae; Dynamic Optimization; State-space model; Bang-bang control.

1. INTRODUCTION

Algal biomass production is receiving considerable attention because of the potential for the production of valuable chemicals for food supplements as well as lipids that can be a source of sustainable bio-fuels. Most studies on algae focus on the physiology of biomass and valuable substance production, and on the design of suitable bio-reactors (Barbosa et al. 2005; Bosma 2010; Cuaresma et al. 2009; Janssen et al. 2003; Richmond and Cheng-Wu 2001). The control aspects of photo-bioreactors (PBRs) have received much less attention, and focussed on pH control (Berenguel et al. 2004) or CO_2 supply (Buehner et al. 2009) .

Over the past years, considerable efforts have been devoted to the development of an optimal control strategy for crop production in greenhouses (van Straten et al., in press, 2010) This consists essentially out of two hierarchical steps: (i) dynamic optimization, to be performed with smooth, assumed nominal weather, and (ii) on-line receding horizon optimal control. The dynamic optimization delivers a co-state trajectory of the biomass, which represents the marginal value of an extra unit of biomass at any time. It can subsequently be used to derive suitable on-line control laws. The approach requires (i) a dynamical state space model of the system, (ii) the formulation of an suitable (economic) goal function, (iii) the forecast of the external variables (weather), (iv) a suitable solution method.

This paper explores the applicability of this methodology for control of a flat plate PBR. We first derive a model, formulate a goal function, and then solve the control problem for two sample light patterns over 3 days. Numerical results as well as a preliminary analysis are presented that shed light on the optimal control problem of the PBR. Finally, similarities and difference with the greenhouse case are briefly discussed, suggesting directions for further investigations.

2. THE PBR BIOMASS MODEL

This paper focuses on the biomass production. At this stage, the generation of the final chemical valuables, which constitutes a more elaborate control problem, is not yet explored. It is assumed that supply of CO_2 and nutrients are non-limiting, and that surplus dissolved oxygen is removed with a rate that is sufficient to prevent growth inhibition. Temperature and pH are assumed to be ideally controlled at a level considered optimal for the algal type studied. The reactor consists of vertically placed transparent sheets in which the biomass broth is contained, ideally mixed by air flow agitation. The reactor receives time varying diffusive and direct sunlight on both sides. A light model is used to derive the photon flux towards the algae as a function of measured direct and diffuse sunlight. Light irradiance inside the reactor is attenuating towards the centre according to Lambert-Beer

$$I(z,t) = I_o(t)e^{-\varepsilon(t)z} + I_d(t)e^{-\varepsilon(t)(d-z)} \tag{1}$$

where $I_o(t)$ and $I_d(t)$ are the incident radiation through the vertical flat plates in μmol[phot]$s^{-1}m^{-2}$ at time t at $z = 0$ and $z = d$, respectively, with d the distance (in m) between the plates, and ε (t) the extinction coefficient (m^{-1}), given by the self-shading equation

$$\varepsilon(t) = \varepsilon_o + \alpha C_X(t) \tag{2}$$

in which ε_o is the extinction coefficient of the cultivation medium (m^{-1}), C_X is the algal biomass (g[dw]m^{-3}), and α the absorption coefficient (m^2g^{-1}[dw]). The dependency of the growth rate is modeled according to the Steele equation

$$f\left(I(z,t)\right) = \frac{I(z,t)}{I_s}\exp\left(1-\frac{I(z,t)}{I_s}\right) \tag{3}$$

where I_s is the irradiance at which the growth rate is maximal. Define the effective growth rate (d^{-1}) at temperature T as

$$\mu_T\left(I_o(t), I_d(t), C_X(t)\right) = \mu_{\max}\frac{1}{hd}\int_{z=0}^{d}\int_{x=0}^{h} f_P\left\{I\left[x,z,t,C_X(t)\right],T_{sp}\right\}dxdz \tag{4}$$

where x is the vertical dimension to height h and z the horizontal dimension to thickness d. Defining the maintenance rate (d^{-1}) at temperature T as k_{mT}, then the algal biomass dynamics is given by

$$\frac{dC_X(t)}{dt} = \left(\mu_T(I_o(t), I_d(t), C_X(t)) - k_{mT} - D(t)\right)C_X(t) \tag{5}$$

where $D(t)$ is the dilution rate (m^3[water]m^3[rv]d^{-1}; rv stands for reactor volume). Note that (4) integrates the growth rate over the horizontal optical path and over the height of the reactor. In this paper, for simplicity, the vertical light distribution (relevant for direct light and shading) is encapsulated in the calculation of a representative uniform irradiance at the surface. The simple one-state model (5) does not describe any adaptation of the parameters to prolonged light regimes.

The model is parameterized on data from Bosma et al., (2007) for the alga *Monodus subterraneus*. In particular Fig. 1 shows the behaviour of the Steele equation, viz. some data at the optimal temperature 23.5 °C, and the polynomial curve presented by Bosma. The parameter values used in all calculations are $d = 0.02$ m, $\mu_{\max T} = 0.9\ d^{-1}$, $k_{mT} = 0.1\ d^{-1}$, $I_s = 350$ μmol[phot]$s^{-1}m^{-2}$, $\varepsilon_o = 2\ m^{-1}$, $\alpha = 0.15\ m^2g^{-1}$[dw]. It is nice to note that the saturation light value coincides with values derived from production experiments for 'warm' algae in Lake Balaton (van Straten and Herodek, 1982).

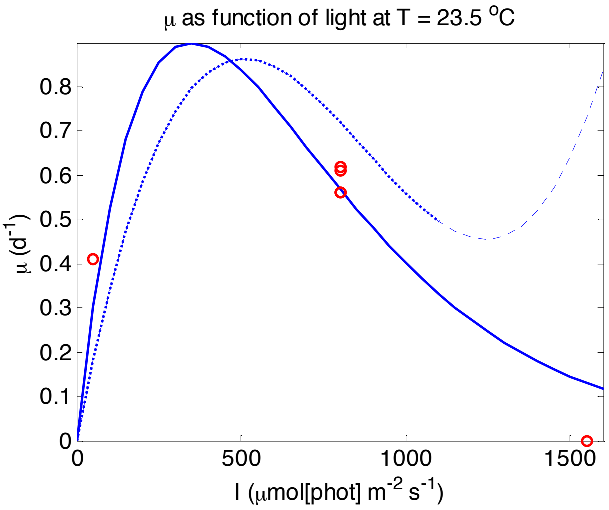

Fig. 1. Parameterization of the Steele function (solid line), compared to data ('o') and polynomial approximation.

3. THE OPTIMAL CONTROL PROBLEM

By defining the state $x = C_X$, control input $u = D$, external inputs $v = [I_o, I_d]^T$, and the parameters p as above, (5) can be written in standard state space form

$$\dot{x} = f(x,u,v,p) \tag{6}$$

with f given by obvious substitutions in the right hand side of (5). A possible goal for the operation of the PBR is to maximize volumetric productivity, which leads to the goal function

$$J = \int_{t_o}^{t_f} DC_X dt \tag{7}$$

or

$$J = \int_{t_o}^{t_f} L(x,u)dt \quad \text{with } L = ux \tag{8}$$

The optimal control problem is to find the optimal control trajectory $u^*(t)$ that maximizes J subject to the control constraint,

$$0 \le u(t) \le 2 \tag{9}$$

determined by the maximum dilution rate (set at 2 d^{-1} here). The computation of the trajectory requires the specification of a nominal light trajectory. Case A is a repetitive three day pattern, shown in the graphs below, and case B assumes that the irradiance on the second day is only half of standard.

4. RESULTS

Numerical calculations were done in Matlab with the tomlab/propt toolbox (Tomlab Optimization AB, Västerås, Sweden). All trajectories are approximated by polynomials