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Stability Analysis of System with Fixed Structure Controller for Industrial Multi-Stage Separation Process via Vector Lyapunov Function

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Abstract: The present paper is devoted to the evaluation of stability analysis of multi-stage separation process (multicomponent distillation) control system based on the tray-by-tray nonlinear dynamic process model. The vector Lyapunov function (LF) jointly with comparison functions (CF) are proposed for the stated problem solution. The overall system of differential equations is decomposed on the subsystems so that the LF for each subsystem can be determined. It was shown that the unforced subsystems of distillation model are unstable. But the stabilizing effect of composite system is reached in a natural way by the interaction among these subsystems. This gives the possibility to formulate stability conditions using CF. The estimation of stability domain for multivariable fixed-structure composition (temperature) controller for two-product industrial distillation column is given as illustrative example.

Keywords: nonlinear models, multivariable control systems, decomposition, stability analysis, Lyapunov function, distillation columns.

1. INTRODUCTION

The stable distillation control is the significant problem in the chemical industry. The considerable amount of works (Skogestad, 1997) are devoted to the synthesis of such controllers which provide a robust stability of control system (Christen et. al., 1997). It is well known that the distillation model has a large dimensionality. Usually, the model linearization is fulfilled in the neighborhood of the desired operating point. After that, the nonlinear properties are taking into account as uncertainty for the development of the μ- or H_∞-optimal controllers. However, this approach may give the crude results because the linear models are considered.

The goal of this paper consists in the evaluation of stability analysis of multicomponent distillation control system based on the rigorous tray-by-tray nonlinear dynamic process model. There are few works in this line of investigation (Coffey et. al, 2001; Rosenbrock, 1963). In the current work the author proposes to use the decomposition feature of the distillation model. This gives a possibility to consider the dynamic properties of each individual separation stage (as subsystem) and to operate with reduced order systems of differential equations based upon the vectors of LF and CF (Matrosov, 1962; Bellman, 1962).

The survey of vector LF methods applications in the work of Lakshmikantham et. al., 1991 shows that the significant problems are established by CF determination for each subsystem. In this connection, the Radial Basis Functions (RBF) neural nets are applied for more exact CF design. It should be noted that an issue of the current results can be realized for stability margin estimation of an industrial distillation column control systems in the frame of commercial software package.

2. STATEMENT OF PROBLEM

The general process arrangement is depicted on the Fig. 1. The following assumptions are held for examined dynamic distillation model:

- constant molar vapor flows;
- tray hydraulics is described by Francis weir formula.

Assume that the top (T_1) and bottom (T_N) temperatures are stabilized by the known (fixed) structure decentralized controllers. The widespread controllers in practice are PI-controllers. Hence, the control system is described by the following set of differential equations:

$$\dot{L}_j = \frac{V_{j+1} + L_{j-1} - V_j - L_j}{\tau_{h_j}}\ ; j=1,\ldots,N, \qquad (1)$$

$$\dot{x}_{ji} = \frac{V_{j+1}y_{j+1,i} + L_{j-1}x_{j-1,i} - V_j y_{ji} - x_{ji}(V_{j+1} + L_{j-1} - V_j)}{U_j}\ ;$$
$$j=1,...,N;\ i=1,...,c, \qquad (2)$$

$$\dot{T}_j = -\frac{1}{\theta(x_{ji}, K_{ji})}\sum_i K_{ji}\dot{x}_{ji}\ ; j=1,\ldots,N, \qquad (3)$$

$$\dot{V}_1 = \dot{T}_1 R_1 + (T_1^{sp} - T_1)\frac{R_1}{\tau_1}\,, \qquad (4)$$

$$\dot{V}_N = \dot{T}_N R_N + (T_N^{sp} - T_N)\frac{R_N}{\tau_N}\,, \qquad (5)$$

where $\theta(x_{ji}, K_{ji}) = \sum_i x_{ji} \dfrac{dK_{ji}}{dT_{ji}}$.

The equations (1)-(5) are often used for distillation towers simulation and reflect the basic physical-chemical sense of the process. If the highly non-ideal mixtures are separated then the equations (1)-(5) can be easy modified by the substitution of the appropriate vapor-liquid equilibrium model without loss of the system structure.

It is necessary to estimate the values of the controller parameters wherein the plant asymptotic stability is guaranteed.

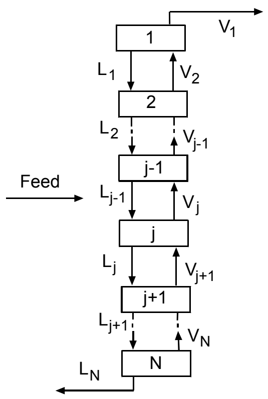

Fig.1 Separation stages interconnection

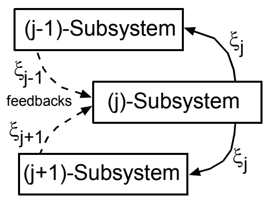

Fig.2 Subsystems interactions

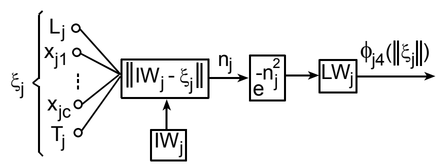

Fig.3 Comparison function representation by RBF-neuron

3. DISTILLATION CONTROL SYSTEM MODEL DECOMPOSITION

One way to solve the stability analysis problem of large-scale dynamic system consists in its decomposition on the small dimensionality subsystems for which it is easy to infer the stability conditions. After that, the composite system examination can be performed (Grujic and Siljak, 1973). The distillation column contains N interconnected separation stages. The individual stage can be considered as subsystem S_j from physical point of view (Fig.2). The dynamic behavior of S_j is described by the state vector ξ_j

$$\dot{\xi}_j = g_j(\xi_j) + h_j(\xi),\ j=1,\ldots,N \tag{6}$$

The ξ_j is an element of the composite state vector ξ of the overall system S.

$$\xi = [\xi_1^{\mathrm{T}}\xi_2^{\mathrm{T}}\ldots\xi_N^{\mathrm{T}}]^{\mathrm{T}}. \tag{7}$$

The function g_j in (6) corresponds to the subsystem S_j itself and the function h_j represents the action of the composite system S on the subsystem S_j. If $h_j \equiv 0$ then we have unforced subsystem. By applying decomposition technique (6)-(7) to initial system (1)-(5) the various right parts of eq.(6) can be obtained for each S_j. In order to simplify the further analysis assume that $U_j=U=const$ (tray holdup) and $\tau_{hj}=\tau=const$ (tray hydraulic time constant).

Thus, for j=1 and $\xi_1 = [L_1\ x_{11} \cdots x_{1c}\ T_1\ V_1]^{\mathrm{T}}$ the following form of (6) can be derived

$$g_1(\xi_1) = \begin{bmatrix} \dfrac{V_F - V_j - L_j}{\tau_h} \\ -\dfrac{x_{j1}V_F}{U_j} \\ \vdots \\ -\dfrac{x_{jc}V_F}{U_j} \\ \dfrac{1}{\theta(x_{ji},K_{ji})}\dfrac{V_F}{U_j} \\ -\dfrac{R_1}{\theta(x_{ji},K_{ji})}\dfrac{V_F}{U_j} + (T_j^{sp} - T_j)\dfrac{R_1}{\tau_1} \end{bmatrix};$$

$$h_1(\xi) = \begin{bmatrix} \dfrac{V_N}{\tau_h} \\ \dfrac{V_N(y_{j+1,1} - x_{j1}) + V_F y_{j+1,1}}{U_j} \\ \vdots \\ \dfrac{V_N(y_{j+1,c} - x_{jc}) + V_F y_{j+1,c}}{U_j} \\ -\dfrac{1}{\theta(x_{ji},K_{ji})}\sum_i K_{ji}\left(\dfrac{V_N(y_{j+1,i} - x_{ji}) + V_F y_{j+1,i}}{U_j}\right) \\ \dfrac{R_1}{\theta(x_{ji},K_{ji})}\sum_i K_{ji}\left(\dfrac{V_N(y_{j+1,i} - x_{ji}) + V_F y_{j+1,i}}{U_j}\right) \end{bmatrix}. \tag{8}$$

For j=2,…,f-2 and $\xi_j = [L_j\ x_{j1} \cdots x_{jc}\ T_j]^{\mathrm{T}}$

$$g_j(\xi_j) = \begin{bmatrix} -\dfrac{L_j}{\tau_h} \\ -\dfrac{y_{j1}V_F}{U_j} \\ \vdots \\ -\dfrac{y_{jc}V_F}{U_j} \\ \dfrac{1}{\theta(x_{ji},K_{ji})}\sum_i K_{ji}\dfrac{V_F}{U_j}y_{ji} \end{bmatrix};$$