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Optimal Industrial Controller Tuning Algorithms in View of Constraints for Stability Margins ⋆

Igor B. Yadykin * **Michael M. Tchaikovsky** *

* *Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia (e-mail: jad@ipu.ru, mmtchaikovsky@hotmail.com).*

Abstract: This paper presents a technique for optimal $\mathcal{H}_2$ tuning of a fixed-order and fixed-structure controller for SISO plant represented by a state-space realization. The proposed technique is based on minimization of some $\mathcal{H}_2$ proximity criterion for transfer functions of closed-loop control system and its implicit reference model (internal model control) subject to restrictions onto $\mathcal{H}_\infty$ norm of transfer function of closed-loop system guaranteeing its stability. Proposed tuning algorithm for the controller parameters uses estimates of the plant parameters obtained via parametric identification. It is shown that necessary conditions for minimum of $\mathcal{H}_2$ norms of open and closed-loop systems coincide with necessary conditions for minimum of Frobenius norm of a matrix tuning polynomial linearly depending on the controller parameters. The proposed technique is illustrated by a numerical example.

Keywords: Fixed-order controller, optimal tuning, internal model control.

1. INTRODUCTION

The problem of fixed-order and fixed-structure controller tuning has been known for more than half a century starting from Ziegler and Nichols (1942) as one of the classic problems of the control theory. Great number of papers and several monographs are devoted to this problem (see e.g. Astrom and Hagglund (2006)), Datta et al. (2000). Analytic methods based on information on structure and form of plant mathematical model play the main role among the methods for solving this problem. These include:

- tuning methods based on solving the controller design problem comprising LMI-based methods;
- automatic tuning methods based on application of relay feedback;
- methods based on indirect adaptive control, or implicit reference model (internal model control).

For recent two decades, many papers devoted to application of powerful $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimization tools to design and tuning problems for fixed-structure controllers have been presented, e.g. by Balandin and Kogan (2007), Bao et al. (1999), Genc (2000), McFarlane and Glover (1992), Polyak and Scherbackov (2003), Tan et al. (2002), Zhou et al. (1996).

In paper of McFarlane and Glover (1992), a practically effective solution for fixed-order controller tuning problem was obtained. It is based on shaping frequency responses of open control loop by means of pre- and post-filters (loop shaping) in conjunction with minimizing $\mathcal{H}_\infty$ norm of closed-loop system. The controller tuning problem is close to the plant identification problem that implies application of constrained and unconstrained optimization technique for finding optimal controller tuning algorithms in model matching problem (see e.g. Morari and Zafiriou (1989), Poznyak (1991)) and, in particular, in internal model control.

2. PROBLEM STATEMENT

Consider a linear continuous time invariant control system consisting of the dynamic plant and fixed-order controller

$$\begin{bmatrix} \dot{x}_p(t) \\ \hline y(t) \end{bmatrix} = \left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & 0 \end{array}\right] \begin{bmatrix} x_p(t) \\ \hline u(t) \end{bmatrix}, \quad (1)$$

$$\begin{bmatrix} \dot{x}_c(t) \\ \hline u(t) \end{bmatrix} = \left[\begin{array}{c|c} A_{cm} & B_c \\ \hline C_{cm} & D_c \end{array}\right] \begin{bmatrix} x_c(t) \\ \hline g(t) - y(t) \end{bmatrix}, \quad (2)$$

where $x_p(t) \in \mathbb{R}^{n_p}$ is the plant state, $y(t) \in \mathbb{R}^1$ is the plant output, $u(t) \in \mathbb{R}^1$ is the control, $x_c(t) \in \mathbb{R}^{n_c}$ is the controller state, $g(t) \in \mathbb{R}^1$ is the reference signal, and the matrices $A_p, B_p, C_p, A_{cm}, B_c, C_{cm}$, and D_c have appropriate dimensions. Assume that plant (1) is completely controllable and observable, the state-space realizations (A_p, B_p, C_p) and (A_c, B_c, C_c, D_c) are minimal, and the matrices A_p, B_p, and C_p are known or can be defined via parametric identification. Also assume $g(t) \in \mathcal{L}_2[0, +\infty)$. We are interested in tracking the reference input for arbitrary set of plant parameters inside of some bounded region Σ. It is assumed that controller (2) has fixed structure. The feature of controller tuning consists in that the controller structure can not be changed in course of tuning procedure, i.e. the given matrices A_{cm} and C_{cm} are fixed, and only the elements of the matrix B_c and scalar value D_c are to be adjusted. Such situation appears, for instance, when controller (2) is a PID controller. Denote the generalized tuning vector $G \triangleq [\, B_c \ D_c \,]$. The goal of

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the considered controller tuning problem based on the constancy principle for internal model of control loop consists in reaching the identity

$$y_m(t) \equiv y(t), \tag{3}$$

where $y_m(t)$ is the output of implicit (virtual) reference model of system (1)–(2) under condition that the plant input is fed by test signal $g(t)$, and the plant parameters belong to some admissible and bounded set

$$\Sigma = \{A_p, B_p, C_p : \underline{a}_{pij} \leqslant a_{pij} \leqslant \overline{a}_{pij}, \underline{b}_{pi} \leqslant b_{pi} \leqslant \overline{b}_{pi}, \underline{c}_{pj} \leqslant c_{pj} \leqslant \overline{c}_{pj}\}.$$

The implicit reference model for system (1)–(2) is given by

$$\begin{bmatrix} \dot{x}_{pm}(t) \\ \hline y_m(t) \end{bmatrix} = \begin{bmatrix} A_{pm} & B_{pm} \\ \hline C_{pm} & 0 \end{bmatrix} \begin{bmatrix} x_{pm}(t) \\ \hline u_m(t) \end{bmatrix}, \tag{4}$$

$$\begin{bmatrix} \dot{x}_{cm}(t) \\ \hline u_m(t) \end{bmatrix} = \begin{bmatrix} A_{cm} & B_{cm} \\ \hline C_{cm} & D_{cm} \end{bmatrix} \begin{bmatrix} x_{cm}(t) \\ \hline g(t) - y_m(t) \end{bmatrix}, \tag{5}$$

where the state vectors of plant and controller, as well as the plant output and control vectors have the same dimensions as similar vectors in system (1)–(2). The controller tuning procedure after the plant identification has been done consists of two stages:

- synthesis of the controller parameters for nominal mode;
- optimal controller tuning according to given tuning criterion.

At that, it is assumed that the plant parameters at zero time take on any constant values from the admissible set Σ.

The goal condition (3) in frequency domain under assumption of zero initial conditions is given by the identities

$$\Phi(j\omega) \equiv \Phi_m(j\omega) \quad \forall \omega \in (-\infty, +\infty), \tag{6}$$

$$W(j\omega) \equiv W_m(j\omega) \quad \forall \omega \in (-\infty, +\infty), \tag{7}$$

where $W(s)$ and $\Phi(s) = \frac{W(s)}{1+W(s)}$ are the transfer functions of open- and closed-loop systems, $W_m(s)$ and $\Phi_m(s) = \frac{W_m(s)}{1+W_m(s)}$ are that of open- and closed-loop reference model, correspondingly. For nominal mode we have $W(s) = W_m(s)$. Let us pass from the identity of transfer functions to the identity of polynomials generated by these transfer functions. This corresponds to the following polynomial controller tuning equation:

$$\begin{aligned} &C_p(sI - A_p)^{-1} B_p C_{cm}(sI - A_{cm})^{-1} B_c \\ &\qquad + C_p(sI - A_p)^{-1} B_p D_c \\ &= C_{pm}(sI - A_{pm})^{-1} B_{pm} C_{cm}(sI - A_{cm})^{-1} B_{cm} \\ &\qquad + C_{pm}(sI - A_{pm})^{-1} B_{pm} D_{cm}. \end{aligned}$$

Applying series expansion of resolvents in left- and right-hand parts of the last equality and multiplying its both parts to the product of characteristic polynomials of the plant, controller, and implicit reference plant and controller models, we obtain the following equality of the matrix polynomials

$$\begin{aligned} &P_1 s^{2n_c+n_p-1} + \cdots + P_{2n_c+n_p-1} s + P_{2n_c+n_p} \\ &= N_1 s^{2n_c+n_p-1} + \cdots + N_{2n_c+n_p-1} s + N_{2n_c+n_p}. \end{aligned} \tag{8}$$

Define the adaptability matrices

$$L = [\, L_{\mu 1} \;\; L_{\mu 2} \,], \quad N^{\mathrm{T}} = [\, N_{\mu 1}^{\mathrm{T}} \;\; N_{\mu 2}^{\mathrm{T}} \,]$$

and the tuning functional J_1 as follows

$$J_1 = \sum_{\mu=0}^{2n_c+n_p-1} \operatorname{tr} (P_\mu - N_\mu)^{\mathrm{T}} (P_\mu - N_\mu),$$

$$P_\mu = \sum_{\eta=1}^{2} L_{\mu\eta} G_\eta, \qquad N_\mu = N_{\mu 1} + N_{\mu 2},$$

$$L_{\mu 1} = \sum_{\sigma=0}^{n_c} \sum_{i=j}^{n_c-1} \sum_{\eta=\nu}^{n_p-1} a_{m\sigma} a_{cm\eta+1} a_{j+1} C_p A_p^{i-j} B_p C_{cm} A_{cm},$$

$$L_{\mu 2} = \sum_{\sigma=0}^{n_c} \sum_{i=j}^{n_c-1} \sum_{\nu=0}^{n_p} a_{m\sigma} a_{cm\nu} a_{i+1} C_p A_p^{i-j} B_p,$$

$$\forall \sigma, \nu, j : \sigma + \nu + j = \mu,$$

$$N_{\mu 1} = \sum_{\sigma=0}^{n_c} \sum_{i=j}^{n_c-1} \sum_{\eta=\nu}^{n_p-1} a_\sigma a_{cm\eta+1} a_{i+1} C_{pm} A_{pm}^{i-j} B_{pm} C_{cm} A_{cm}^{\eta-\nu} B_{cm},$$

$$N_{\mu 2} = \sum_{\sigma=0}^{n_c} \sum_{i=j}^{n_c-1} \sum_{\nu=0}^{n_p} a_\sigma a_{cm\nu} a_{i+1} C_{pm} A_{pm}^{i-j} B_{pm} D_{cm},$$

$$\forall \sigma, \nu, j : \sigma + \nu + j = \mu,$$

where a_i, a_{mi}, a_{cmi} are the coefficients of the characteristic polynomials of the plant, implicit reference model of plant and controller, respectively.

To find the minimum of the functional J_1, one need solve the equivalent matrix equation

$$LG = N \tag{9}$$

in the matrix $G = \left[\, B_c^{\mathrm{T}} \;\; D_c \,\right]^{\mathrm{T}}$ (see Yadykin (1985))

The matrix G_{opt} minimizing the functional J_1 is the solution to this equation:

$$G_{\text{opt}} = L^{\dagger} N, \tag{10}$$

where $L^{\dagger}$ denotes the Moor-Penrose generalized inverse of the matrix L.

Identity (8) can be rewritten as

$$\begin{aligned} \frac{M_c(j\omega) M_p(j\omega)}{Q_c(j\omega) Q_p(j\omega)} &= \frac{M_o(j\omega)}{Q_o(j\omega)} \\ &\equiv \frac{M_{om}(j\omega)}{Q_{om}(j\omega)} = \frac{M_{cm}(j\omega) M_{cm}(j\omega)}{Q_{cm}(j\omega) Q_{cm}(j\omega)} \\ &\qquad \forall \omega \in (-\infty, +\infty), \end{aligned} \tag{11}$$

where $M_o(s), M_c(s), M_p(s)$ are the numerator polynomials of transfer functions of the open-loop system, controller, and plant, $Q_o(s), Q_c(s), Q_p(s)$ are the denominator polynomials of these transfer functions, repectively. Denote

$$\begin{aligned} &M_o(j\omega) Q_{om}(j\omega) \triangleq P_o(s), \\ &\qquad M_{om}(j\omega) Q_o(j\omega) \triangleq Q_o(s), \\ &\qquad\qquad F_o(s) \triangleq P_o(s) - Q_o(s), \end{aligned} \tag{12}$$

$$F_o(s) = \sum_{i=0}^{2n_c+n_p-1} (P_i - N_i) s^i.$$

Define the following controller tuning functional