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Estimation of Multi-components System's reliability: Comparison of two Graphical Model Approaches

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Abstract: Reliability analysis is an integral part of system design and operating. Moreover, it can be an input to optimize maintenance policies. Recently, Bayesian Networks (BN) and Dynamic Bayesian Networks (DBN) have been proved relevant to represent complex systems and perform reliability studies. The major drawback of this approach comes from the constraint on the state sojourn times which are necessarily exponentially distributed, as in usual markovian approaches. Therefore, a new formalism was introduced to avoid this constraint: the Graphical Duration Models (GDM). This paper aims to quantify the reliability estimation error due to an exponential approximation when the system follows other kinds of sojourn time's distributions. Finally results obtained by DBN and GDM will be compared.

Keywords: Reliability, Markov models, Semi-Markov process, Probabilistic Graphical Models, Bayesian Network.

1. INTRODUCTION

Reliability analysis has become an integral part of system design and operating. This is especially true for systems performing critical applications such as mass transportation systems. Typically, the results of such analysis are given as inputs to a decision support tool in order to optimize the maintenance operations.

Unfortunately, in most of cases, the state of the system cannot be exactly predicted. Indeed, in most of practical applications, deterministic approach of degradation process and transitions towards failure state is uncommon. This is one of the reasons which have led to the important development of probabilistic methods in reliability. A wide range of works about reliability analysis is available in the literature. For instance in numerous applications, the aim is to model a multi-states system and hence to capture how the system state behaves over time or after a given number of solicitations. This problematic can be partially solved using the Markov framework. Aven and al. (1999).

More recently, some studies involving the use of Bayesian Networks (BN) have been proved relevant to represent complex systems and perform reliability studies. For instance, Boudali and al. (2005) show how to model complex system dependability by means of BN. In Langseth and al. (2007), the authors explain how dynamic fault trees can be represented by BN. Finally, other works experiments Dynamic Bayesian Networks (DBN) to study the reliability of a dynamic system represented by a Markov chain. Weber and al. (2006).

The major drawback of this approach comes from the constraint on the state sojourn times which are necessarily exponentially distributed. Indeed, if the sojourn times are far from an exponential distribution, a markovian modelling will be unable to take this fact into account. The degradation process modelling will be therefore biased. In a reliability analysis, such a mistake can have strong consequences, especially if one wants to optimize parameters of maintenance policies based on reliability.

This issue can be overcome by the use of semi-Markov models which allow considering any kind of sojourn time distributions. Limnios and al. (2001). The main interests of an approach based on graphical models come from their intuitive use and the numerous existing processing tools (e.g. inference and learning methods). In previous works, an original graphical model was introduced to capture the behaviour of complex stochastic degradation processes, allowing any kind of state sojourn distributions as well as an accurate context description: the Graphical Duration Models (GDM). Donat and al. (2008a), Bouillaut and al. (2008).

The present study aims to quantify the reliability's estimation error due to an exponential approximation (by use of a markovian modelling) when the system follows other kinds of sojourn time's distributions. Finally, results obtained by the two approaches (DBN and GDM) will be compared.

This paper is therefore divided into four sections. Section 2 briefly describes the formalisms of DBN and GDM. Then, section 3 introduces a toy system, commonly used in the reliability field, consisting in three components series-parallel. A comparison of reliability analysis results, obtained for both modelling, will be done. Finally, some conclusions and perspectives are discussed in section 4.

2. RELIABILITY MODELLING TOOLS

2.1 Markovian approach: BN modelling a Markov Chain

BN are mathematical tools relying on both the probability theory and the graph theory. Jensen (1996). They allow to qualitatively and quantitatively representing uncertainty. Basically, BN are used to compactly describe the joint distribution of a collection of random variables $\boldsymbol{X}=(X_1,\ldots,X_N)$ taking their values in $\boldsymbol{\mathcal{X}} = \{\mathcal{X}_1\ldots\mathcal{X}_M\}$.

Formally, a BN denoted by $\mathcal{M}$ is defined as a pair $(\mathcal{G},\{p_n\}_{1\leq n\leq N})$ where $\mathcal{G}=(\xi, \varepsilon)$ is a Directed Acyclic Graph (DAG) and $\{p_n\}_{1\leq n\leq N}$ a set of Conditional Probability Distributions (CPD) associated with the random variables. These distributions aim to quantify the local stochastic behaviour of each variable. The graph nodes and the associated random variables being both represented by $\xi = \{X_1, \ldots, X_N\}$. ε is the set of edges encoding the conditional independence relationships among these variables. Finally, $\mathcal{G}$ is said to be the qualitative description of the BN.

Besides, both the qualitative (i.e. $\mathcal{G}$) and quantitative (i.e. $\{p_n\}$) parts can be automatically learnt, if some complete or incomplete data or experts opinions are available. Jordan (1999).

Using BN is also particularly interesting because of the easiness for knowledge propagation through the network. Indeed, various inference algorithms allow computing the marginal distribution of any subset of variables. The most classical one relies on the use of a junction tree. Lauritzen and al. (1988).

Finally, note that such modelling is able to represent dynamic systems (e.g. which contain variables with time dependant distributions) via the DBN solution. Murphy (2002).

A DBN is a convenient extension of the BN formalism to represent discrete sequential systems. Indeed, DBN are dedicated to model data which is sequentially generated by some complex mechanisms (time-series data, bio-sequences, number of mechanical solicitations before failure…). It is therefore frequently used to model Markov chains. Fig. 1 illustrates this property, introducing a DBN modelling the Markov Chain of the sequence $\boldsymbol{X}=(X_1, \ldots, X_N)$ taking its values in the set $\boldsymbol{\mathcal{X}}$. This DBN is described by the probabilities that quantify the transitions from one state of $\boldsymbol{\mathcal{X}}$ to another.

Fig. 1. DBN modelling a Markov Chain.

More precisely, a DBN defines the probability distribution of a collection of random variables $(X_t)_{t\geq 1} = (X_{1,t}, \ldots, X_{D,t})_{t\geq 1}$ where t is the discrete time index. In this study, we consider only a special class of DBNs, called 2-slice Temporal Bayesian Networks (2-TBN). A 2-TBN is a DBN which satisfies the Markov property of order 1, that is, the future of the studied stochastic process is independent from its past given its present. This property is formally denoted by:

$X_{t-1} \perp\!\!\!\perp X_{t+1} \mid X_t$ for all $t \geq 2$.

A pair of BNs $(\mathcal{M}_1, \mathcal{M}_\rightarrow)$ allows to defined a 2-TBNs. BN $\mathcal{M}_1$ represents the joint distribution of the initial process $X_1=(X_{1,1}, \ldots, X_{D,1})$. Then, the distribution $P(X_1)$ admits the following factorisation:

$$P(X_1) = P\left(X_{1,1},\ldots,x_{D,1}\right) = \prod_{d=1}^{D} P(X_{D,1} \left| X_{pa_{d,1}}\right.)$$

Where $pa_{d,1}$ is the indices sequence of the parents of the d-th variable in slice 1.

BN $\mathcal{M}_\rightarrow$ defines the transition model, i.e. the distribution of X_t givenX_{t-1}. In this case the BN factorisation leads to:

$$P(X_t \left| X_{t-1}\right.) = P\left(X_{1,t},\ldots,X_{D,t} \left| X_{1,t-1},\ldots,X_{D,t-1}\right.\right) = \prod_{d=1}^{D} P(X_{d,t} \left| X_{pa_{d,t}}\right.)$$

Where $pa_{d,t}$ denotes the indices sequence of the parents of $X_{d,t}$ which can only stand in slices t and t − 1 according to the Markov property of order 1. Thereby, the joint distribution of the random variables over a temporal horizon T, i.e. $(X_t)_{1\leq t\leq T}$, can be found out by simply "unrolling" the 2-TBN until a sequence of length T is obtained:

$$P\left((X_t)_{1\leq t\leq T}\right) = \prod_{d=1}^{D} P(X_{d,1} \left| X_{pa_{d,1}}\right.).\prod_{t=1}^{T}\prod_{d=1}^{D} P(X_{d,t} \left| x_{pa_{d,t}}\right.)$$

Besides considering 2-TBNs as a pair of two static BNs allows to easily derive the previously mentioned learning procedures to both initial and transition models. On the other hand the inference problem in 2-TBNs suffers from an important increase of time and space complexity. Specific algorithms have been proposed to partially resolve the issue. Boyen and al. (1998), Cox (1972).

In reliability analysis, one can be interested in modelling how a system changes from an *up* state to a *down* state over time. Most of the time, a modelling based on the DBN formalism was done. Boudali and al. (2005), Weber and al. (2006), Langseth and al. (2007)…

The major drawback of this approach comes from the constraint on the state sojourn times which are necessarily exponentially distributed. Indeed, if the considered system follows (or is very close to) an exponentially distributed degradation process, this approach can be perfectly suitable. On the other hand, if the sojourn times are far from an exponential distribution, a markovian modelling will be unable to take this fact into account and the modelling of the degradation process will be biased. In a reliability analysis, such inaccurate estimation can have strong consequences,