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Estimations of an absolute error and the scheme of the approached solution problems of the scheduling theory ⋆

Alexander A. Lazarev *

* *Institute of Control Sciences of the Russian Academy of Sciences, Profsoyuznaya st. 65, 117997, Moscow, Russia (e-mail: jobmath@mail.ru)*

Abstract: For single and multi-machine scheduling problems with the criterion of minimization maximum lateness the metrics ρ has been used for the first time. A theorem of estimating the absolute error has been proved. The idea of the offered approach consists in construction by an initial instance of a problem of other instance for which it is possible to find the optimum or approximated solution, with the minimal distance up to an initial instance in entered metric.

Keywords: Scheduling algorithms; Estimation Parameters; Model Approximation; Multi-processor Scheduling; Theoretical Scheduling; Estimation Absolute Error.

1. INTRODUCTION

We are given a set $N = \{1, \ldots, n\}$ of n jobs that must be processed on m machines $M = \{1, \ldots, m\}$. Preemption of the jobs is not allowed. Each machine can handle only one job at a time. For each job j we have: r_j – release date; $0 \le p_{ji} \le +\infty$ – processing time job j on the machine i (if $p_{ji} = +\infty$, then job j can not process on the machine i); d_j – due date. Between jobs ratios of a precedence in the form of an acyclic oriented graph $G \subset N \times N$ are set. Through π_i we will definete the schedule of the jobs processeded on the machine $i, i = 1, \ldots, m$. Naturally, admissible schedules without artificial idle times of the machines, satisfying the graph are considered only.

In this paper, we consider the approach finding of the approximate solution with the guaranteed absolute error for the problems minimizing maximum lateness. The idea of the approach consists in construction to a initial instance A such instance B (with the same number of jobs) with minimum of estimation of absolute error that

$$0 \le L^A_{max}(\pi^B) - L^A_{max}(\pi^A) \le \rho_d(A,B) + \rho_r(A,B) + \rho_p(A,B),$$

where

$$\rho_d(A,B) = \max_{j\in N}\{d^A_j - d^B_j\} - \min_{j\in N}\{d^A_j - d^B_j\},$$

$$\rho_r(A,B) = \max_{j\in N}\{r^A_j - r^B_j\} - \min_{j\in N}\{r^A_j - r^B_j\}$$

and

$$\rho_p(A,B) = \sum_{j\in N} |p^A_j - p^B_j|,$$

and π^A, π^B – optimal schedules for instances A and B, respectively. Besides $\rho(A,B) = \rho_d(A,B) + \rho_r(A,B) + \rho_p(A,B)$ satisfies to properties of the metrics in $(3n-2)$-dimensional space $\{(r_j, p_j, d_j) \mid j \in N\}$ with fixed in two parameters.

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A schedule π is uniquely determined by a permutation of the elements of N, which consists of m schedules π_i for each machine $i, i = 1, \ldots, m, \pi = \bigcup_{i=1}^{m} \pi_i$. The objective function is maximum lateness $L_{max}(\pi) = \max_{j\in N} L_j(\pi)$, where $L_j(\pi) = C_j(\pi) - d_j$, and $C_j(\pi)$ is complete time job $j \in N$ in schedule π.

This problem $P|prec; r_j|L_{max}$ is a generalisation of some NP-hard problems, for example: $P|intree; r_j; p_j = 1|C_{max}$, $P|outtree; p_j = 1|L_{max}$ Brucker et. al. (1977); $P2|chains|C_{max}$ Du et. al. (1991); $P||C_{max}$ Garey and Johnson (1978); $1|r_j|L_{max}$, $P2||C_{max}$ Lenstra et. al. (1977); $P|prec; p_j = 1|C_{max}$ Ullman (1975.

For some related problems exist polynomially solvable cases: $P2|prec; r_j; p_j = 1|L_{max}$ Garey and Johnson (1977); $P|p_j = p; r_j|L_{max}$ Simons (1983); $1|prec; pmtn; r_j|L_{max}$ Blazewicz (1976); $P|chains; r_j; p_j = 1|L_{max}$ Baptiste et. al. (2004); $1|prec; pmtn; r_j|L_{max}$ Baker et. al (1983); $P|chains; r_j; p_j = 1|L_{max}$ Dror et. al. (1998); $P2|prec; p_j = p|L_{max}$ Garey and Johnson (1976); $JMPM|prec; r_j; n = 2|L_{max}$ Jurisch (1995); $1|prec|L_{max}$ Lawler (1973); $1|prec; p_j = p; r_j|L_{max}$ Simons (1978).

Estimation of an absolute error for the NP-hard problem minimizing maximum lateness for single machine $1|r_j|L_{max}$ has been considered in Lazarev (1989, 2009), Lazarev et. al. (2006).

2. DEFINITIONS

We denote by $L^A_j(\pi)$ and $C^A_j(\pi)$ lateness and complete time of job j in schedule π for instance A with parameters $\{G^A, (r^A_j, p^A_j, d^A_j) \mid j \in N\}$. And, accordingly, $L^A_{max}(\pi) = \max_{j\in N} L^A_j(\pi)$ and π^A– optimal schedule for instance A.

Let's call an instance $B = \{G^B, (r^B_j, p^B_j, d^B_j) \mid j \in N\}$ **inverse** to initial instance $A = \{G^A, (r^A_j, p^A_j, d^A_j) \mid j \in N\}$,

if
$$r_j^B = -d_j^A,\ p_j^B = p_j^A,\ d_j^B = -r_j^A,\ \forall\, j \in N.$$
In instance B orientation of all edges of the graph is replaced on opposite, $\overleftarrow{G}^B = \overrightarrow{G}^A$. And schedule $\pi_i' = (j_{n_i}, j_{n_i-1}, \ldots, j_1)$ is **inverse** to schedule $\pi_i = (j_1, \ldots, j_{n_i-1}, j_{n_i})$ for each machine $i \in M$.

For two any instances A and B we'll define following functions:
$$\begin{cases} \rho_d(A,B) = \max_{j\in N}\{d_j^A - d_j^B\} - \min_{j\in N}\{d_j^A - d_j^B\}, \\ \rho_r(A,B) = \max_{j\in N}\{r_j^A - r_j^B\} - \min_{j\in N}\{r_j^A - r_j^B\}, \\ \rho_p(A,B) = \sum_{j\in N}\left(\max_{i\in M}(p_{ji}^A - p_{ji}^B)_+ + \max_{i\in M}(p_{ji}^A - p_{ji}^B)_-\right), \\ \rho(A,B) = \rho_d(A,B) + \rho_r(A,B) + \rho_p(A,B), \end{cases} \quad (1)$$
where $(x)_+ = \begin{cases} x, x>0, \\ 0, x\le 0; \end{cases}$; $(x)_- = \begin{cases} -x, x<0, \\ 0, x\ge 0; \end{cases}$; $|x| = (x)_+ + (x)_-$.

The "processing part" $\rho_p(A,B)$ can be written down on another:
$$\rho_p(A,B) = \sum_{j\in N}\left(\max_{i\in M}\{(p_{ji}^A - p_{ji}^B), 0\} - \min_{i\in M}\{(p_{ji}^A - p_{ji}^B), 0\}\right).$$

As for instances $A = \{G, (r_j, p_j, d_j) \,|\, j \in N\}$ and $A' = \{G, (r_j+\alpha, p_j, d_j+\beta) \,|\, j \in N\}$ the set of optimum schedules equals, it is possible "to fix" two parameters, for example, $\alpha = -r_1$ and $\beta = -d_1$. Then the function $\rho(A,B)$ satisfies to properties of normed metrics in $(3n-2)$-dimensional space. Let's say that instances A and B be equivalent, if there exist constants d and r
$$d_j^A = d_j^B + d, \quad r_j^A = r_j^B + r, \quad p_j^A = p_j^B \qquad \forall j \in N.$$
Obviously for equivalent instances the set of optimal schedules equals. Set of nonequivalent instances we will define through $\mathcal{L}_n$.

3. ESTIMATION OF ABSOLUTE ERROR

Lemma 1. Let $A = \{G^A, (r_j, p_j, d_j^A) | \, j \in N\}$ and $B = \{G^B, (r_j, p_j, d_j^B) | \, j \in N\}$ (with identical release and processing times $r_j, p_j, j \in N$,) be two instances then for any schedule π holds
$$L_{max}^B(\pi) - L_{max}^A(\pi) \le \max_{j\in N}\{d_j^A - d_j^B\}. \quad (2)$$

Proof. For any $j \in N$ we have: $L_{max}^A(\pi) + \max_{i\in N}\{d_i^A - d_i^B\} \ge C_j(\pi) - d_j^A + d_j^A - d_j^B = C_j(\pi) - d_j^B$. So, $L_{max}^A(\pi) + \max_{i\in N}\{d_i^A - d_i^B\} \ge \max_{j\in N}\{C_j(\pi) - d_j^B\} = L_{max}^B(\pi)$. □

The instances A and B are "symmetric", so obviously for any schedule π holds
$$L_{max}^A(\pi) - L_{max}^B(\pi) \le \max_{j\in N}\{d_j^B - d_j^A\}. \quad (3)$$

Lemma 2. Let $A = \{G, (r_j, p_j, d_j^A) | \, j \in N\}$ and $B = \{G, (r_j, p_j, d_j^B) | \, j \in N\}$ (with identical release and processing times $r_j, p_j, j \in N$, and graph G) be two any instances then
$$0 \le L_{max}^A(\pi^B) - L_{max}^A(\pi^A) \le \rho_d(A,B).$$

Proof. From (2), (3) for schedules π^A and π^B we have
$$L_{max}^A(\pi^A) + \max_{j\in N}\{d_j^A - d_j^B\} \ge L_{max}^B(\pi^A), \quad (4)$$
$$L_{max}^B(\pi^B) + \max_{j\in N}\{d_j^B - d_j^A\} \ge L_{max}^A(\pi^B). \quad (5)$$
Schedule π^B is optimal for instance B so
$$L_{max}^B(\pi^A) \ge L_{max}^B(\pi^B). \quad (6)$$
From (4)–(6)
$$L_{max}^A(\pi^A) + \max_{j\in N}\{d_j^A - d_j^B\} \ge L_{max}^A(\pi^B) - \max_{j\in N}\{d_j^B - d_j^A\},$$
then
$$L_{max}^A(\pi^A) + \rho_d(A,B) \ge L_{max}^A(\pi^B) \ge L_{max}^A(\pi^A).$$
□

The instances A and B are "symmetric" so obviously $0 \le L_{max}^B(\pi^A) - L_{max}^B(\pi^B) \le \rho_d(A,B) = \rho_d(B,A)$.

Lemma 3. Let A and B be inverse instances and π and π' – inverse schedules, then $L_{\max}^A(\pi^A) = L_{\max}^B(\pi^B)$.

Lemma 4. Let $A = \{G^A, (r_j^A, p_j, d_j^A) | \, j \in N\}$ and $B = \{G^B, (r_j^B, p_j, d_j^B) | \, j \in N\}$ (with identical processing times $p_j, j \in N$,) be two any instances then
$$0 \le L_{max}^A(\pi^B) - L_{max}^A(\pi^A) \le \rho_r(A,B). \quad (7)$$

Lemma 5. Let $A = \{G, (r_j, p_j^A, d_j) | \, j \in N\}$ and $B = \{G, (r_j, p_j^B, d_j) | \, j \in N\}$ (with identical release times and due dates $r_j, d_j, j \in N$, and graph preceding G) be two any instances then
$$0 \le L_{max}^A(\pi^B) - L_{max}^A(\pi^A) \le \sum_{j\in N}|p_j^A - p_j^B| = \rho_r(A,B). \quad (8)$$

Theorem 6. Let $A = \{G, (r_j^A, p_j^A, d_j^A) | \, j \in N\}$ and $B = \{G, (r_j^B, p_j^B, d_j^B) | \, j \in N\}$ (with identical graph preceding G) be two any instances then
$$0 \le L_{max}^A(\pi^B) - L_{max}^A(\pi^A) \le \rho(A,B). \quad (9)$$

From "symmetric" the instances A and B holds
$$0 \le L_{max}^B(\pi^A) - L_{max}^B(\pi^B) \le \rho(A,B) = \rho(B,A). \quad (10)$$

Theorem 7. Let $A = \{G, (r_j^A, p_j^A, d_j^A) | \, j \in N\}$ and $B = \{G, (r_j^B, p_j^B, d_j^B) | \, j \in N\}$ (with identical graph preceding G) be two any instances then
$$0 \le L_{max}^A(\overline{\pi}) - L_{max}^A(\pi^A) \le \delta^B(\overline{\pi}) + \rho(A,B), \quad (11)$$
where $\delta^B(\overline{\pi}) = L_{max}^B(\overline{\pi}) - L_{max}^B(\pi^B), \overline{\pi}$ is approximate schedule.

4. THE SCHEME OF APPROACHED DECISION OF THE PROBLEM

The idea finding approximated solution of the problem consists of two stages. On the first step to the initial instance $A = \{G, (r_j^A, p_j^A, d_j^A) | \, j \in N\}$ is such change of its parameters r_j, p_j and d_j that the obtained instance $B = \{G, (r_j^B, p_j^B, d_j^B) | \, j \in N\}$ belongs to a set polynomially solvable instances of the initial problem. On the next step we'll find optimal schedule to instance B. According to Theorem 6 the schedule π^B to initial instance A have $0 \le L_{max}^A(\pi^B) - L_{max}^A(\pi^A) \le \rho(A,B)$.