Reliability Engineering and System Safety 180 (2018) 104–110

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Optimal allocation of reliability improvement target based on the failure risk and improvement cost

Kyungmee O. Kim[a], Ming J. Zuo[*,b,c]

[a] Department of Industrial Engineering, Konkuk University, Seoul 143-701, South Korea
[b] School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China
[c] Department of Mechanical Engineering, University of Alberta, Edmonton T6G2G8, Canada

ARTICLE INFO

Keywords:
Failure severity
Allocation weight
System improvement target
Failure mode and effects analysis (FMEA)

ABSTRACT

In this paper, we consider the problem of assigning a given reliability improvement target of an existing series system to its constituent subsystems in view of the failure risk and improvement cost. Previous research has solved this problem by developing an allocation weight under the assumption that the failure risk and improvement cost are independent, and by improving every subsystem in proportion to the allocation weight. Differently, we develop an optimization model to maximize the profit from reliability improvement for the given reliability improvement target. The profit is derived from the functional relationship between the failure risk and improvement cost. The optimal solution shows that not all subsystems are improved, and the priority of subsystem improvement is determined by the difference between the failure severity and the rate of increase in the improvement cost. A numerical example is given to illustrate the advantage of the proposed method over the previous method.

1. Introduction

During a given stage of system development, an engineer allocates the requirement for system performance to individual subsystems in such a manner that the efforts of the subsystems for achieving their apportionments are well-balanced. The system performance is often specified in a form of the reliability, failure probability, or failure rate. The corresponding problem of assigning the system requirement to subsystems is called reliability allocation [17].

Two approaches are taken in the literature for studying reliability allocation. The first one regards reliability allocation as a problem of multiple criteria decision making in which an allocation weight is used as a proportionality factor to assign the failure rate requirements to subsystems [4,11,17,19,27]. The second approach considers a reliability optimization problem in which the development cost is minimized subject to a reliability constraint [9,10,25], or the system reliability is maximized subject to resource constraints [21], under the assumption that the relationships between the reliability and resources are known.

Recently, the subsystem failure risk appeared as a criterion for determining the allocation weight. Early research [16,28,29] calculated the subsystem failure risk by the arithmetic average of risk values of its all possible failure modes, assuming that the risk of each failure mode is calculated through failure mode and effect analysis (FMEA) as the product of the failure occurrence and the failure severity [7,12]. Then, an allocation weight was expressed in terms of the subsystem failure risk for assigning a low failure rate to a subsystem having a high risk. However, it is infeasible to achieve a lower failure rate for a subsystem expected to fail more frequently. Therefore, Kim et al. [18] proposed a new allocation weight by considering only the subsystem severity rather than the subsystem risk. If their method is applied to improve an existing system rather than develop a new system, then the allocated subsystem failure rate may be higher than the current subsystem failure rate, as the failure occurrence is not considered while determining the allocation weight. Subsequently, Yadav and Zhuang [30] revised the allocation weight of Kim et al. [18] to consider the improvement cost in addition to the failure severity. They were the first to use the allocation weight for allocating the reliability improvement target of an existing system rather than the reliability requirement of a new system.

In this paper, we reconsider the problem of assigning the reliability improvement target of an existing series system in view of the failure risk and the improvement cost. In Section 2, we reveal that the method of Yadav and Zhuang [30] improves every subsystem in proportion to the allocation weight, and therefore a negative failure rate is allocated if a current subsystem has a low failure rate but high severity, or if the system improvement target is large. To solve this problem differently,

* Corresponding author.
E-mail address: ming.zuo@ualberta.ca (M.J. Zuo).

https://doi.org/10.1016/j.ress.2018.06.024
Received 22 September 2017; Received in revised form 23 June 2018; Accepted 30 June 2018
Available online 06 July 2018
0951-8320/

we present an optimization model in Section 3 to maximize the profit of a given reliability improvement task. The profit is defined by a trade-off between the failure risk reduction and the improvement cost. Then, the optimal solution is obtained from the Kuhn–Tucker conditions. In Section 4, a numerical example is given to compare the proposed method with the previous method. Finally, conclusions are given in Section 5.

2. Literature review

In this section, we review the previous risk-based allocation weights after explaining how the allocation weight is used for reliability allocation.

2.1. Allocation weight

Consider a system composed of k independent subsystems in series. Let $\overline{F}^*$ be the system reliability requirement for a fixed mission time. Let $\overline{F}_i^*$ denote the reliability apportioned to subsystem i for $i = 1, \ldots, k$. In the literature [13,17,19], the allocated subsystem reliability is determined by

$$\overline{F}_i^* = (\overline{F}^*)^{w_i}, \quad i = 1, \ldots, k, \tag{1}$$

where w_i denotes the allocation weight for subsystem i with $0 < w_i < 1$ and $\sum_{i=1}^{k} w_i = 1$. Assuming that each subsystem has a constant failure rate, Eq. (1) is reduced to [17,30]

$$\lambda_i^* = w_i \lambda^*, \quad i = 1, \ldots, k, \tag{2}$$

where λ^* denotes the system failure rate requirement and λ_i^* is the failure rate assigned to subsystem i. Then, $\sum_{i=1}^{k} \lambda_i^* = \lambda^*$ holds. Eq. (2) shows that the allocated failure rate is directly proportional to the corresponding allocation weight, and thus a higher failure rate is apportioned to a subsystem having a larger allocation weight [19].

To determine w_i in Eqs. (1) and (2), an engineer selects one or more criteria which are independent of each other. Then, each subsystem is evaluated for every criterion, and the contributions from different criteria are aggregrated using an operator such as addition or multiplication to obtain the score of each subsystem, assuming that the functional relationship between the criteria and the subsystem reliability is not known. Finally, the normalized subsystem score is used as the allocation weight [9,11,17,27]. That is, the allocation weight is obtained by

$$w_i = \frac{g_i}{\sum_{i=1}^{k} g_i}, \quad i = 1, \ldots, k, \tag{3}$$

where g_i is the score of subsystem i determined from the selected criteria. Over the last several decades, various allocation weights have been developed considering different criteria such as the subsystem intricacy [4,9,27], state-of-the-art technology [9,17], mission time [17,19], environmental conditions [4,9], and functionality [11]. Further research has been done for considering the importance of each criterion relative to the others [4,27], incorporating the subjectivity in evaluating different criteria [4,26], or employing different operators to aggregate the contributions of several criteria [6,22].

2.2. Risk-based allocation weight

Recently, the risk of subsystem failure appeared as a criterion for determining the allocation weight. We explain how the risk is calculated through FMEA before reviewing the previous risk-based allocation weights.

FMEA is an effective method used during a given stage of a system life cycle to identify all independent failure modes possible for each subsystem and investigate the consequences of the occurrences of failure modes [8,9,12]. Let N_i be the number of failure modes possible for subsystem i for $i = 1, \ldots, k$. Let R_{ij} be the risk of the jth failure mode of subsystem i. By a formal definition of risk [7,15,18],

$$R_{ij} = O_{ij} \times S_{ij}, \quad i = 1, \ldots, k, \quad j = 1, \ldots, N_i, \tag{4}$$

where O_{ij} is the failure probability or the failure rate that measures how likely the jth failure mode of subsystem i is to occur during the given stage, and S_{ij} is the severity to evaluate how serious the consequence would be in the eventuality that the jth failure mode of subsystem i occurs. In manufacturing industries, S_{ij} is further expressed by the product of the detection and severity under the assumption that there is negligible loss, i.e., zero severity, if the failure mode occurrence is detected before it reaches the customer at the next stage of the system life cycle [12,14].

Suppose that no quantitative data are available for evaluating O_{ij} and S_{ij} in Eq. (4). Then, a set of ten linguistic expressions is shown to the expert to pick a statement that best describes each of the failure occurrence and severity for the jth failure mode of subsystem i for all $i = 1, \ldots, k$ and $j = 1, \ldots, N_i$ [7,12]. Then, a ten-point numerical value is assigned to each statement for scoring the qualitative response of the expert; the corresponding value of R_{ij} in Eq. (4) is called the risk priority number (RPN). Although the RPN has been widely used in traditional FMEA, many authors have emphasized that the RPN calculation is subject to limitations because the ten-point numerical values are ordinal values that cannot be multiplied together [5,14]. Subsequently, various cost-based FMEA models have been developed in the literature [2,3,20,24] to measure S_{ij} in financial terms depending on whether and when the failure mode occurrence is detected.

To consider the failure risk as a criterion in reliability allocation, early research [16,28,29] determined w_i in Eq. (3) using

$$g_i = 1 - \frac{R_i}{\sum_{i=1}^{k} R_i}, \quad i = 1, \ldots, k, \tag{5}$$

where R_i denotes the risk of subsystem i calculated by

$$R_i = \frac{1}{N_i} \sum_{j=1}^{N_i} R_{ij}, \quad i = 1, \ldots, k,$$

assuming that R_{ij} in Eq. (4) is obtained from the RPN value. If R_i in Eq. (5) increases, then w_i in Eq. (3) decreases. This implies that if O_{ij} gets larger in Eq. (4), then a smaller value of λ_i^* is allocated to subsystem i.

Because the achievement of a low failure rate is technically difficult for a subsystem having a high failure occurrence, Kim et al. [18] proposed a new allocation weight using the failure severity as a criterion. Let S_i be the severity of the consequence when subsystem i has failed. Assuming that the subsystem severity is represented by the severity of the most serious failure mode in a given subsystem, S_i is calculated by

$$S_i = \max(S_{i1}, \ldots, S_{iN_i}), \quad i = 1, \ldots, k, \tag{6}$$

and the allocation weight in Eq. (3) is determined using

$$g_i = (S_i m_i \pi_i)^{-1}, \quad i = 1, \ldots, k, \tag{7}$$

where S_i is calculated as given in Eq. (6), m_i is the number of subsystems in the system having the same value of S_i, and π_i is the relative frequency of the most serious failure mode occurring given that subsystem i results in system failure. As the current subsystem failure rate is not considered in Eq. (7), using this method to improve an existing system rather than design a new system can result in the allocated subsystem failure rate being larger than its current failure rate.

Therefore, Yadav and Zhuang [30] proposed for the first time the use of the allocation weight to assign the system improvement target of an existing system to its subsystems. Let λ be the current failure rate of an existing system and λ_i be the current failure rate of subsystem i for $i = 1, \ldots, k$. Then, $\lambda = \sum_{i=1}^{k} \lambda_i$ holds. Let $\vartriangle$ denote the system improvement target, which is given by $\vartriangle = \lambda - \lambda^* > 0$, where λ^* is the system failure rate requirement. While all of the previous methods [16,18,28,29] employed Eq. (2) to use w_i for distributing λ^*,