Reliability Engineering and System Safety 180 (2018) 189–200

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Multivariate system reliability analysis considering highly nonlinear and dependent safety events

Mohammadkazem Sadoughi[a], Meng Li[a], Chao Hu[a,b,*]

[a] Department of Mechanical Engineering, Iowa State University, Ames, Iowa 50011, USA
[b] Department of Electrical and Computer Engineering, Iowa State University, Ames, Iowa 50011, USA

ARTICLE INFO

Keywords:
Multivariate Gaussian process
Multivariate probability of improvement
System limit-state function
System reliability analysis

ABSTRACT

Most of the existing system reliability analysis methods have focused on series and parallel systems whose components are weakly nonlinear and dependent. This paper proposes a new system reliability analysis method, named multivariate system reliability analysis (MSRA), for complex engineered systems with highly nonlinear and dependent components that are connected in series, parallel, and mixed configurations. The proposed method first employs multivariate Gaussian process (MGP) to sequentially construct a single surrogate jointly over the performance functions of all components and then performs Monte Carlo simulation on the surrogate model for system reliability analysis. The joint surrogate is updated adaptively using a novel acquisition function named multivariate probability of improvement (MPI). MGP considers the correlations between the component performance functions and provides a joint Gaussian prediction of these functions. This joint Gaussian surrogate model allows the use of MPI to achieve a dynamic trade-off between exploring the regions in the input space with high prediction uncertainty and exploring those that are close to the system limit-state function. The results of three abstract and two practical case studies show that MSRA is capable of achieving better accuracy in estimating the system reliability than the existing surrogate-based methods.

1. Introduction

In engineering system design, a desired feature of the designed system is that it can maintain a high reliability level. Given various sources of uncertainty, reliability is usually expressed as a percentage, which measures the probability that an engineered system or its components will function properly under stated conditions. The importance of reliability has been well conceived by engineers and researchers in the past few decades. Considerable advances have been made in the fields of reliability analysis [1, 2] and reliability-based design optimization (RBDO) [3, 4].

Despite the considerable advances in reliability analysis, scaling a reliability analysis methodology from the component to system level has been difficult. A complex engineered system can have multiple failure modes that are often attributed to multiple physical or cyber components. Each of these failure (or safety) modes can be modeled as a component safety event (or simply component). Reliability analysis of the engineered system simultaneously considering these component safety events is termed system reliability analysis [5]. This is in contrast to component reliability analysis where only a single component safety event is considered. Due to the great importance of system reliability analysis in analyzing and improving the reliability of a complex system, its technical development has drawn considerable attention from researchers in various engineering fields, such as reliability engineering, engineering design, civil and structural engineering, and cyber-physical systems. The search for efficient and accurate ways for system reliability analysis has resulted in the development of a variety of methods that can be generally categorized as (i) bound-based approaches, (ii) analytical approaches, and (iii) surrogate-based approaches.

Bound-based approaches estimate the system reliability level by providing the lower and upper bounds. The first-order bound (FOB) method [6] estimates the lower and upper bounds of the system reliability by assuming all the component safety events as mutually independent or completely dependent. This method is simple to use but often generates too wide range of system reliability. To improve the precision of the system reliability estimate, Ditlevsen [7] proposed the so-called second-order bound (SOB) method that considers the interactions between any two component safety events. The SOB method generally gives much tighter bounds compared with the FOB method. However, both FOB and SOB have limited utility due to the ignorance of the higher-order interactions among component safety events and the lack of smoothness in the bounds from resulting from the

* Corresponding author at: Department of Mechanical Engineering, Iowa State University, Ames, Iowa 50011, USA.
E-mail addresses: chaohu@iastate.edu, huchaostu@gmail.com (C. Hu).

https://doi.org/10.1016/j.ress.2018.07.015
Received 21 February 2018; Received in revised form 9 June 2018; Accepted 11 July 2018
Available online 19 July 2018

Nomenclature

CE:	convergence estimator
MGP:	multivariate Gaussian process
MPI:	multivariate probability of improvement
SLSF:	system limit-state function
n_c:	number of components
G_c:	component performance function
$\mathbf{x}$:	vector of random input variables
$\Omega_{\mathbf{x}}^{\mathrm{CS}}$:	component safety region in the **x** space
$\Omega_{\mathbf{x}}^{\mathrm{SS}}$:	system safety region in the **x** space
$\Omega_{\mathbf{G}}^{\mathrm{CS}}$:	component safety region in the **G** space
$\Omega_{\mathbf{G}}^{\mathrm{SS}}$:	system safety region in the **G** space
$\Omega_{\mathbf{G}}^{\mathrm{SLSF}}$:	probable region of the system LSF in the **G** space
$\mathcal{D}$:	data set used to build MGP
$n_{\mathcal{D}}$:	number of known sample points in $\mathcal{D}$
$f_{\mathbf{x}}(\mathbf{x})$:	joint probability density function of **x**
n_{MCS}:	number of Monte Carlo simulation (MCS) random samples
$\mathbf{Y}$:	stack vector in MGP to collect all the known inputs and outputs
$\mathbf{\Sigma}_{\mathbf{Y}}$:	covariance matrix in MGP
$\boldsymbol{\theta}_c$:	hyper-parameters in MGP with respect to the *c*th performance function

minimization/ maximization terms involved in the bound formulae. To address these limitations, Song and Der Kiureghian [8] formulated system reliability as a Linear Programming (LP) problem, referred to as the LP bounds method. The LP bounds method is able to calculate optimal bounds for system reliability based on the probabilities of mutually exclusive and collectively exhaustive events. However, the performance of the LP bounds method is extremely sensitive to the accuracy of the probability approximation for the mutually exclusive and collectively exhaustive events. More studies on the bound-based methods for system reliability analysis can be found in Refs. [9,10]. Despite the development of the mentioned bound-based methods, bounds are usually not convenient in practice, since they often provide a wide range of system reliability estimate.

Analytical approaches try to derive an explicit formula of system reliability based on the component safety events. Upon the derivation of the explicit formula, commonly used component reliability analysis methods, such as most probable point (MPP) based method [11] and dimension reduction method (DRM) [12], can be conveniently utilized for system reliability assessment. Youn et al. [13] proposed the complementary intersection method (CIM), which provides an explicit formula for system reliability analysis by defining the complementary intersection (CI) event. With the definition of CI event, the CI-matrix was introduced and utilized to solve the reliability of a series system with any component reliability analysis method. However, the CIM is not directly applicable to parallel and mixed systems. To address this problem, Wang et al. [14] proposed a generalized CIM (GCIM) framework that enables the application of the CIM to analyze the reliability of an engineered system with any configuration. The system structure matrix is proposed to characterize any system configuration in a comprehensive manner. The binary decision diagram technique is employed to identify a system's mutually exclusive path sets, of which each path set is a series system. Consequentially, system reliability with any system configuration is decomposed into the probabilities of the mutually exclusive path sets, which can be evaluated using any component reliability analysis methods. Although the CIM/GCIM provides an explicit formula for system reliability analysis, accurate estimation of high order CI-events remains a persistent challenge.

Another popular way to perform system reliability analysis is to use surrogate-based approaches. These approaches integrate surrogate modeling techniques with Monte Carlo simulation (MCS). Many surrogate modeling techniques have been applied to component reliability analysis and these techniques include dimension reduction methods (DRM) [12], stochastic spectral methods [15], stochastic collocation methods [16], and Kriging-based methods [17, 18]. Kriging-based methods build the surrogate model over a performance function following a sequential process, in which an initial surrogate model is first constructed with an initial set of observations (e.g., at a set of sample points generated by the Latin hypercube sampling (LHS) technique), and then continuously refined by identifying and adding new sample points (often one new point at each iteration) [19, 20]. The ability to adaptively identify the important regions in the input space and locally refine the surrogate model in these regions is a key advantage of these methods when applied to system reliability analysis. After the surrogate model is built, MCS is most often performed as a post-processing step to assess the component reliability. Several researchers have applied Kriging-based surrogate modeling to system reliability analysis. Based on the efficient global reliability analysis (EGRA) method [5], Bichon et al. [21] proposed the use of a composite Gaussian process model for system reliability analysis. Their work adopted the so-called composite expected feasibility function as the acquisition function to find the new samples located in the component limit-state regions that contribute the most to the system failure. However, this sampling procedure spans the whole input space and efforts may be wasted in regions carrying low probability content. Similarly, Wang and Wang [19] introduced the so-called integrated performance measure approach (iPMA) for system reliability, in which an integrated performance measure function is derived to envelop all component safety events. iPMA adopts a comprehensive system-level sampling rule to handle multiple failure modes concurrently for efficient system reliability assessment. However, the sampling strategy of iPMA is only based on the prediction error of the Kriging model for each component without considering the importance regions to choose the sample points. To identify important regions as well as recognize unimportant component(s) during sequential sampling, Fauriat and Gayton [20] developed the Active learning and Kriging-based SYStem reliability (AK-SYS) method via the combined use of Active learning and Kriging-based MCS. However, the efficiency of AK-SYS may decrease in cases where large differences in magnitude exist among the component responses, because in such cases, AK-SYS may fail to efficiently track the important regions close to the system limit-state. Recently, Yang et al. [22] introduced the concepts of adaptive truncating region (ATR) and truncated candidate region (TCR) and applied the concepts to active learning Kriging (ALK) model for system reliability analysis, which is abbreviated as ALK-TCR. ALK-TCR greatly decreases the probability of adding training points in the unimportant regions and unimportant component(s) by identifying the TCR. However, TCR mainly focuses on the system failure region, which may also include regions far away from the system limit-state. Furthermore, all the aforementioned methods are only applicable to series and parallel systems and cannot be directly applied to estimate the reliability of mixed systems. More recently, Hu et al. [23] introduced an efficient Kriging surrogate modeling approach for system reliability analysis. Their approach constructs composite Kriging surrogates through selection of singular value decomposition (SVD)-based and individually constructed Kriging models, with an aim to combine the advantages of both types of Kriging model. This approach can deal with any type of system and the SVD-based Kriging models are capable of capturing the correlations between multiple component responses. However, individual Kriging models still need to be constructed, which may result in the lack of consideration of the dependence between components.

Most of the aforementioned Kriging-based system reliability analysis methods build separate and independent Kriging models for component performance functions. However, if the multiple component