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Improved inverse Gaussian process and bootstrap: Degradation and reliability metrics

Jingbo Guo[a], Changxi Wang[a], Javier Cabrera[b], Elsayed A. Elsayed[a,*]

[a] Department of Industrial and Systems Engineering, Rutgers University, Piscataway, NJ 08854, USA
[b] Department of Statistics, Rutgers University, Piscataway, NJ 08854, USA

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ABSTRACT

The inverse Gaussian (IG) process is commonly used in modeling monotonically increasing degradation processes. Traditional degradation modeling considers the process parameters as functions of time and environmental conditions. However, in many practical situations, the degradation increment in the next time interval may depend on degradation state at the current time interval. Therefore, in this paper, we propose an improved inverse Gaussian (IIG) process which considers the dependency between degradation increments and prior degradation states. Reliability metrics of the IIG process are estimated and validated using crack length growth data as well as simulated degradation data. Results show that the proposed model provides more accurate reliability metrics than the IG process model. Bootstrap of degradation increments or detrended degradation increments is introduced as a supplementary method to estimate the remaining life probability interval.

1. Introduction

Most systems and components deteriorate over time and fail when the degradation accumulates beyond an acceptable level, which is usually called the threshold. Compared with traditional failure-time data analysis, which aims to predict remaining life based on failure times of components and systems, degradation analysis aims to capture the underlying failure process with limited test samples and degradation increments data [1].

The general degradation path method is widely used in degradation analysis [1–4]. A two-stage procedure is proposed: degradation path selection and parameter estimation. In practice, stochastic processes are generally used to account for the inherent random degradation over time. Three classes of stochastic degradation processes have been developed based on the assumption of the accumulation of degradation with time, they are Wiener process with drift [5], Gamma process [6] and IG process [7].

The Wiener process with drift has been extensively studied in degradation modeling due to its mathematical properties and physical interpretations [8–12]. In the Wiener process with drift model, the degradation increments are assumed to be independent and normally distributed, with mean and variance dependent on time. Due to the ease of incorporating explanatory variables in the Wiener process models, many observable environmental factors such as temperature and humidity, are considered [5, 13]. Meanwhile, the unobservable factors such as field use conditions and the detecting ability of sensors can also be considered in the model [14, 15]. One advantage of Wiener process with drift is that the first passage time can be easily obtained [16–18]. Moreover, hitting times of bivariate Wiener process models have also been studied [19]. However, a distinct feature of Wiener process with drift model is that the sample path is not necessarily monotonic, which limits its application in monotonically increasing degradation such as crack and corrosion growth modeling. Alternatively, models such as Gamma process and IG process are implemented in these conditions.

Gamma process and its extensions have been widely used as models for degradation or damage of materials. During the last four decades, Gamma processes were satisfactorily fitted to the data of concrete creep [20], fatigue crack growth [21], steel gates corrosion [22], thinning due to corrosion [23, 24], resistors degradation [25] and chloride ingress into concrete [26]. In Gamma processes, increments are independent and non-negative random variables following Gamma distribution with a scale parameter and a time-dependent shape function. The time-dependent shape function is generally monotonically increasing with time. Based on forms of shape functions, the Gamma process models can be classified into two classes, homogeneous Gamma processes and non-homogeneous Gamma processes [27]. Explanatory variables and random effects can be incorporated into the model by designing appropriate shape functions [28]. As extensions of Gamma process models, multivariate Gamma processes have also been applied to cases with multiple degradation indicators [21, 29]. To improve the Gamma

* Corresponding author.
E-mail address: elsayed@soe.rutgers.edu (E.A. Elsayed).

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process model, the dependency between degradation and time is considered in Pandey *et al.* [24]. The dependency of degradation increment in the next moment on current degradation state is also considered [30]. A hierarchical Bayesian-updating Gamma process model is proposed in order to deal with imperfect degradation data [31].

For monotonically increasing degradation data, Gamma processes do not always work well, especially when the degradation increments do not precisely follow Gamma distributions. Under this condition, IG process is proposed as an alternative model. Compared with Gamma processes, the research on IG process as a degradation model is limited. IG process is implemented to model laser devices degradation [32], where Expectation-Maximization (EM) algorithm is used to estimate parameters. Bayesian analysis of IG model is applied to obtain more accurate parameters by continuously updating degradation data [33, 34]. Ye *et al.*[7, 35] study IG models that incorporate explanatory variables which account for heterogeneities and accelerated degradation testing (ADT) planning with IG process model. Similar to the Gamma processes, degradation analysis of systems based on multiple degradation processes is discussed in [29, 36]. In these degradation models, the degradation rate is assumed to be influenced by material, degradation mechanism, environment and other factors.

However, in many cases, the predicted degradation is also affected by the current degradation state. For example, the growth of fatigue crack on an oil pipeline relies not only on the pipe material and environmental factors but also on the starting crack length. The larger the starting crack length, the more probable that it will lead to a higher degradation increment in the next time interval. In this paper, an improved inverse Gaussian (IIG) process model is developed to describe this phenomenon. Rather than only considering the relationship between time and the degradation measurements, the starting degradation state is also utilized to predict the degradation increment in the following time interval.

The bootstrap is a sample-reuse method that is introduced by Efron [37]. It has been used to obtain confidence intervals of statistical parameters. Under the assumption that detrended degradation increments are *i.i.d.*, we propose to use bootstrap as an alternative to parametric degradation models to predict degradation increments by resampling data and finding the probability interval of the remaining life of the unit under degradation monitoring. It works well especially in cases where degradation data are scarce or degradation data do not fit well known parametric distributions. We use fatigue-crack-growth data in Bogdanoff and Kozin [38] and simulated degradation data to motivate our work and verify the applicability of the proposed methods.

2. IIG process and remaining life prediction

2.1. IG process

Suppose that a degradation process $\{y(t), t \geq 0\}$ follows a IG process with scale parameter λ and shape function $\Lambda(t)$ as described in Ye *et al.* [7]. It has the following properties:

1. $y(t)$ has independent increments: $y(t_2) - y(t_1)$ and $y(t_4) - y(t_3)$ are independent of each other for $\forall t_4 > t_3 > t_2 > t_1$
2. Degradation increments follow IG distributions: $(y(t_2) - y(t_1)) \sim IG(\Lambda(t_2) - \Lambda(t_1), \lambda(\Lambda(t_2) - \Lambda(t_1))^2)$ for $\forall t_2 > t_1$

Let $y(t)$ denote the degradation state at time t, $t \geq 0$. $y(0)$ is the starting degradation state. According to the definition of IG process, $y(t) - y(0)$ follows an IG distribution $IG(\Lambda(t) - \Lambda(0), \lambda(\Lambda(t) - \Lambda(0))^2)$. When the shape function $\Lambda(t) = \mu t$, the IG process appropriately describes a fatigue degradation process caused by crack growth [33]. The probability density function (*pdf*) and cumulative distribution function (*cdf*) of $y(t) - y(0)$ are given respectively by:

$$f((y(t)-y(0))|\Lambda(t),\lambda\Lambda^2(t)) = \sqrt{\frac{\lambda\Lambda^2(t)}{2\pi(y(t)-y(0))^3}}\exp\left[-\frac{\lambda((y(t)-y(0))-\Lambda(t))^2}{2(y(t)-y(0))}\right] = \sqrt{\frac{\lambda\mu^2t^2}{2\pi(y(t)-y(0))^3}}\exp\left[-\frac{\lambda((y(t)-y(0))-\mu t)^2}{2(y(t)-y(0))}\right], (y(t)-y(0))>0, \tag{1}$$

$$F((y(t)-y(0))|\Lambda(t),\lambda\Lambda^2(t)) = \Phi\left[\sqrt{\frac{\lambda\mu^2t^2}{(y(t)-y(0))}}\left(\frac{(y(t)-y(0))}{\mu t}-1\right)\right] + \exp(2\lambda\mu t)\Phi\left[-\sqrt{\frac{\lambda\mu^2t^2}{(y(t)-y(0))}}\left(\frac{(y(t)-y(0))}{\mu t}+1\right)\right], \tag{2}$$

The expectation and variance are respectively given by:

$$E((y(t)-y(0))) = \Lambda(t) = \mu t \quad Var((y(t)-y(0))) = \frac{\Lambda(t)}{\lambda} = \frac{\mu t}{\lambda}$$

2.2. IIG process

In this section, we develop an IIG model based on the IG process. Suppose the degradation process $\{y(t), t \geq 0\}$ is observed at every discrete unit of time. Assume that at time t, the degradation state is $y(t)$. The degradation increment $\Delta y(t) = y(t+1) - y(t)$ denotes the degradation during $(t, t+1)$. Rather than modeling this degradation increment with respect to time t, we use the starting crack length $y(t)$ as the reference. In other words, the degradation in a unit time $\Delta y(t)$ follows IG distribution in accordance with the starting crack length $y(t)$. Assume that the shape function takes a linear form as $\Lambda(y(t)) = \mu_0 + \mu_1 y(t)$. The *pdf* of $\Delta y(t)$ is then given by:

$$f(\Delta y(t)|\Lambda(y(t)),\lambda\Lambda^2(y(t))) = \sqrt{\frac{\lambda\Lambda^2(y(t))}{2\pi\Delta y^3(t)}}\exp\left[-\frac{\lambda(\Delta y(t)-\Lambda(y(t)))^2}{2\Delta y(t)}\right], \Delta y(t)>0 = \sqrt{\frac{\lambda(\mu_0+\mu_1 y(t))^2}{2\pi\Delta y^3(t)}}\exp\left[-\frac{\lambda(\Delta y(t)-(\mu_0+\mu_1 y(t)))^2}{2\Delta y(t)}\right], \tag{3}$$

The expectation and variance are given respectively by:

$$E(\Delta y(t)) = \Lambda(y(t)) = \mu_0 + \mu_1 y(t) \quad Var(\Delta y(t)) = \frac{\Lambda(y(t))}{\lambda} = \frac{\mu_0+\mu_1 y(t)}{\lambda}$$

2.3. Remaining life prediction and reliability estimation based on IIG

Suppose the threshold of the degradation process is D. The distribution of remaining life for the IIG process can only be obtained by iterative computation. Assume we have degradation measurements until time t and the degradation measurement at that time is $y(t)$, the remaining life can be obtained by the following procedure:

1. Starting from $i = 1$.
2. Generate a random number $\Delta y(t+i-1)^*$ following the distribution $IG(\Lambda(y(t+i-1)), \lambda\Lambda^2(y(t+i-1)))$. Degradation state now becomes $y(t+i) = y(t+i-1) + \Delta y(t+i-1)^*$
3. Compare $y(t+i)$ to the threshold D. If $y(t+i) < D$, set $i = i+1$, go back to step 2. Else if $y(t+i) \geq D$, the predicted failure time is