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Connectivity evaluation and optimal service centers allocation in repairable linear consecutively connected systems

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ABSTRACT

Many real-world systems in applications such as radio communication, wireless communication, and pipeline transportation can be modeled as linear consecutively connected systems (LCCS) with nodes forming a linear sequence. An LCCS provides a connection between the first and last nodes of the sequence using connecting elements (CEs) located at its nodes. This paper models an LCCS with repairable CEs characterized by different up and down time distributions, and different connection ranges. The distribution parameters of CE down times depend on the location of available service centers (SCs). Thus, the optimal allocation of SCs becomes a relevant and significant optimization problem to formulate and solve for guiding optimal decisions on LCCS maintenance. The objective is to find an SC allocation among predetermined positions that maximizes the expected LCCS connectivity over a specified mission time horizon. To evaluate the objective function of the proposed optimization problem, instantaneous availabilities of repairable CEs with arbitrary up and down time distributions are first determined through a numerical iterative algorithm. Instantaneous and expected LCCS connectivity are then evaluated using a universal generating function method. As demonstrated through examples, the proposed optimization leads to significant improvements in LCCS connectivity and effective allocation and management of maintenance resources.

1. Introduction

Linear consecutively connected systems (LCCSs), originating from consecutive-k-out-of-n: F systems [1–3], have received intensive research attentions in the past few decades due to their abundant applications in diverse industry domains, such as radio communication, sensor monitoring, wireless communication, and pipeline transportation [4,5]. A key feature of LCCSs is a set of system nodes forming a linear, ordered sequence. Connection elements (CEs) with different connection ranges are allocated to these nodes to provide connectivity between the host node and its subsequent nodes along the sequence. They work together to provide a connection between the first (source) and last (destination) nodes of the LCCS, which defines the system connectivity [6].

Following the first introduction of the LCCS model in [4], numerous research efforts have been expended in reliability analysis of binary-state and multi-state LCCSs [4,5,7,8]. A common optimization problem that has been relevant and solved is the optimal CE allocation problem (CEAP) for LCCSs with non-identical CEs. Particularly, CEs can be characterized by different connection rages, and different time-to-failure and time-to-repair distributions. Due to this heterogeneity, different allocations of CEs to the system nodes may lead to great difference in the LCCS performance [9,10]. Actually, it has been shown through many studies that an LCCS performance can be significantly enhanced through the optimal CE allocation [11–13]. Recently, the CEAP was solved for phased-mission LCCSs involving different source and destination nodes in multiple phases [14]. Further common-cause failures were modeled in [15] to consider simultaneous malfunctions of multiple CEs due to the same root cause in the solution to CEAP of phased-mission LCCSs. Extensions were also made to the generic LCCS model by allowing disconnected nodes (gaps) for system functionality. In particular, LCCSs that can tolerate a number of single-node gaps [7,16], a certain size of consecutive gaps [17], or some combined constraint of the former two [18–20] have been modeled and optimized. Another latest development was made to address warm standby redundancy in the LCCS modeling and optimization [21].

The existing works on LCCSs mostly focused on non-repairable CEs; little work has been done for LCCSs with repairable CEs [12,22]. Particularly, in [12] LCCSs subject to preventive replacement and corrective maintenance with constant average repair time were considered. The minimal repair policy was used, which assumes a malfunctioned CE, af-

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Acronyms

cdf	cumulative distribution function
pdf	probability density function
SCAP	service center allocation problem
LCCS	linear consecutively connected systems
ILC	instantaneous LCCS connectivity
CE	connecting element
CEAP	CE allocation problem
SC	service center
UGF	universal generating function

Nomenclature

τ	mission time horizon considered
m	number of discrete time intervals
$a(t)$	instantaneous LCCS connectivity
$A(\tau)$	expected LCCS connectivity over system mission time τ
I	number of LCCS nodes that can host CEs
K	total number of possible locations for SCs
ω	a subset of the set of all the SC positions, $\omega \subset \{1,2,\ldots,K\}$
$\langle T_j, X_j\rangle$	event that the jth failure of a CE occurs at time T_j and the CE spends time X_j in operation mode before the failure
$Q_j(t, x)$	joint distribution function of random values T_j and X_j for a CE
π_i	repair efficiency coefficient of CE located at node i
$J_{i,\omega}$	maximal number of failures of CE located at node i during the time horizon given SCs are located at positions belonging to subset ω
$p_{i,\omega}(t)$	instantaneous availability of CE located at node i under SC allocation ω
$G_i(t)$	random connection range of CE located at node i
g_i	connection range of operating CE located at node i
η_i, β_i	scale, shape parameters of Weibull up time distribution for CE located at node i
$f_i(t)$, $F_i(t)$	*pdf, cdf* of up time for CE located at node i
$\psi_{i,\omega}(t)$, $\Psi_{i,\omega}(t)$	*pdf, cdf* of down time for CE located at node i given SCs are located at positions belonging to subset ω
$D_{i,\omega}$	random down time for CE located at node i given SC are located at positions belonging to subset ω
$d_\omega^{\min}(i)$, $d_\omega^{\max}(i)$	minimal and maximal possible realizations of $D_{i,\omega}$ for CE located at node i
$\mu_\omega(i)$, $\sigma_\omega(i)$	mean and standard deviation of the truncated normal distribution of $D_{i,\omega}$ for CE located at node i
$L_k(t)$	index of the most remote node that can be reached by functioning CEs located at the first k nodes at time t

ter repair, is restored to an "as bad as old" condition. In other words, a repaired CE has an effective age same as that right before the repair. In [22], the CEAP for LCCSs with random repair time and general repair policy was addressed. The general repair policy covers the minimal repair, perfect repair, and imperfect repair [23–25]. In contrast to the minimal repair used in [12], a perfect repair can restore a failed CE to an "as good as new" condition (a repaired CE has an effective age of 0). As an intermediate policy, the imperfect repair can restore a failed CE to a condition between "as bad as old" and "as good as new" [26].

Based on the general repair model of [22], this paper formulates and solves a completely new optimization problem, the optimal service center allocation problem (SCAP) for repairable LCCSs. Repair centers are the places where the equipment and personnel needed to repair failed CEs are located. The down time of failed CEs depends on arrival time of the repair team. Thus, the distance of nodes hosting CEs from the available closest repair service center affects the repair/down time and thus instantaneous availability of a failed CE, and further the connectivity performance of the overall LCCS. The objective of the proposed SCAP is to identify an optimal allocation of service centers among some predetermined locations maximizing the expected system connectivity. Solution to the proposed optimization problem is expected to aid in the optimal decisions on allocating limited maintenance resources for reliable operation of LCCSs.

The remainder of the paper is arranged as follows: Section 2 defines LCCS connectivity measures and presents the system model. Section 3 formulates the SCAP problem addressed in this work. Section 4 presents a numerical iterative algorithm for analyzing instantaneous availability of CEs with random up and down times following arbitrary distributions. Section 5 presents a universal generating function based method for assessing instantaneous and expected system connectivity. Section 6 presents examples to illustrate the proposed optimization. Lastly, Section 7 concludes the paper and presents directions of future research.

2. Description of LCCS model

There are $I+1$ consecutive locations (or nodes) in the considered binary-state LCCS. A connecting element (CE) is allocated at each of the first I nodes to provide a connection between the first (source) node and the $I+1$th (sink) node. Each CE located at node i ($1 \le i \le I$) is characterized by a specific connection range $G_i(t)$, and up and down time distributions. Thus, the most remote node that can be reached by this CE at time t is $i+G_i(t)$. Eq. (1) gives the most remote node that can be reached by the group of CEs located at the first k nodes (i.e., nodes 1, 2,…, k) at time t.

$$L_k(t) = \min\left\{I+1, \max_{1\le i\le k}\{i+G_i(t)\}\right\}. \tag{1}$$

In case of $L_k(t) < k+1$ for any k ($1 \le k < I$), node $k+1$ is disconnected from all the preceding nodes, and thus the source and sink nodes cannot be connected. Therefore, the connectivity condition of the considered LCCS at time t can be given as (2), which returns 1 if the LCCS is connected and 0 otherwise.

$$\phi(G_1(t),\ldots,G_I(t)) = \prod_{k=1}^{I} 1(L_k(t) > k) \tag{2}$$

The instantaneous LCCS connectivity (ILC) at a particular time instant t can be defined as $a(t) = \Pr(a(t) = Pr(\phi(G_1(t),\ldots,G_I(t)) = 1)$. Usually technical systems are planned to operate during specific time horizon after which their elements are replaced and/or structure is changed (due to changes in technology, conditions of functioning and/or system mission). Thus the availability analysis beyond the planned horizon has no sense and the LCCS behavior is modeled within a time horizon τ. The expected LCCS connectivity over a time horizon τ (denoted by $A(\tau)$) can thus be given as

$$A(\tau) = \frac{1}{\tau}\int_0^\tau a(t)dt. \tag{3}$$

There are K possible positions in which service centers (SCs) can be located. Any allocation of SCs can be represented by a subset ω of the set of all the positions: $\omega \subset \{1,2,\ldots,K\}$. The down time distributions of CEs depend on the distance of nodes from the closest SC, which provides repair service to the failed CE. Thus, for any SCs allocation ω, the down time of any CE located at node i is randomly distributed in the interval $[d_\omega^{\min}(i), d_\omega^{\max}(i)]$. The cumulative distribution function (*cdf*) $\Psi_{i,\omega}(t)$ of the random down time is known and such that $\Psi_{i,\omega}(t) = 0$ for $t < d_\omega^{\min}(i)$ and $\Psi_{i,\omega}(t) = 1$ for $t > d_\omega^{\max}(i)$.

The number of repairs experienced by the CE located at node i during time τ cannot exceed $\tau/d_\omega^{\min}(i)$. Thus, the maximal number of failures