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A novel multi-distribution multi-state flow network and its reliability optimization problem

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A R T I C L E I N F O

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A B S T R A C T

The traditional multi-state flow network (MFN) reliability problem is a well-known NP-hard problem, and has been an active area of research for the past four decades. In the MFN reliability problem, the state distribution, i.e. the states and the occurrence probabilities of each arc, is known and fixed. However, the notations used in the MFN problems never considered the state distribution. In addition, each arc has only one state distribution, and this limits the application of the MFN. Thus, the notations relating to the state distribution are added or redefined in this study, and a novel budget-allocation multi-distribution MFN reliability problem is defined and proposed by considering networks with more than one state distribution under different budget allocations. Furthermore, a new algorithm is proposed to solve the proposed novel NP-Hard problem. The correctness and time complexity of the proposed algorithm are analyzed, and one benchmark example is given to demonstrate how to optimize the budget-allocation multi-distribution MFN reliability under different budget constraints.

1. Introduction

Recently, reliability has been widely considered as a crucial factor in various everyday systems, including grid systems [1], repairable systems [2], human systems [3], power transmission and distribution [4], oil/gas production [5], internet of things [6], and logistics system [7], among others [1–25], in which the aim is to design reliable systems and assess their performance. The above real-world systems can all be considered as multistate flow networks (MFNs) which satisfy the Flow Conservation Law [1–10]. There is therefore a continual need for the design of new systems, and improvement or further development of the MFN reliability of numerous systems for engineering, industrial and scientific applications [8].

MFN reliability is a probability that at least a required number of flows, defined as an integer number d, are able to transfer successfully from the source node, i.e. node 1, to the sink node, i.e. node n, under given conditions in the network [8–13].

In MFN reliability, the notation $G(V, E, W)$ is always used to denote an MFN, where $V=\{1, 2, \ldots, n\}$; $E=\{e_1, e_2, \ldots, e_m\}$ and $W=\{W(e_1), W(e_2), \ldots, W(e_m)\}$ are the node set, arc set, and the set of the maximal state of each arc, respectively. For example, Fig. 1 shows a bridge MFN example with $V=\{1, 2, 3, 4\}$, $E=\{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $W=\{W(e_1)=2, W(e_2)=2, W(e_3)=3, W(e_4)=3, W(e_5)=2, W(e_6)=2\}$ based on the states given in Table 1.

The states of each arc are the possible capacity levels, and there is a probability for the occurrence of each state; therefore, the notation represented the MFN must have the state distribution, including the states and their occurrence probabilities, and not just the maximal states of each arc [8–13]. Thus, in this study, the notation $G(V, E, W)$ is redefined to $G(V, E, D)$, where D is a state distribution, e.g., Table 1 gives the state distributions of the MFN in Fig. 1.

In the current MFN reliability problem, only one state distribution is allowed for each arc [1–13]. However, it is more practical for each network to have more than one state distribution if there are such available without violating any budget constraint. This is one of the most popular methods of increasing arc reliability to improve system reliability [14–16]. With a larger budget, the probability of a lower-level state (i.e. state 0) can be reduced, and that of higher-level states can be increased; even the state level can be improved [14–16]. For example, the corresponding state probability of e_1 is 0.1, 0.3 and 0.6 for states 0, 1 and 2 in Table 2, respectively. These values can be improved such that the maximal state is increased to 3, and the new probabilities of the states are 0.1, 0.2, 0.3 and 0.4 for states 0, 1, 2 and 3 (as shown in the second column under e_1), respectively.

To meet the above practical need, a generalized MFN called the Budget-Allocation Multi-Distribution MFN (BDFMN) that incorporates the multi-distribution characteristic is proposed, allowing each arc to have different state distributions for different budgets. Take Fig. 1 as an example, Table 2 is its possible multi-distribution. Note that the val-

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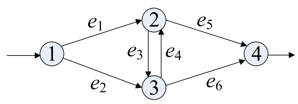

Fig 1. The example multi-state flow network.

Table 1
The state distribution of arcs in Fig. 1.

State\arc	e_1	e_2	e_3	e_4	e_5	e_6
0	.10	.10	.05	.05	.10	.10
1	.30	.40	.25	.20	.40	.30
2	.60	.50	.30	.35	.50	.60
3			.40	.40		

ues of cost in Table 2 comprise the budget to enable better states and probabilities.

The novel budget-allocation multi-distribution MFN problem is a reliability optimization problem [32–35] that aims to solve the optimization reliability of the proposed budget-allocation multi-distribution MFN by selecting the best budget allocations of each arc. There is always a need for engineers to make a decision on which material/component model to use for each system element, with consideration to limited budget, at the design stage or maintenance stage [32–35].

To the best of the authors' knowledge, no study has discussed the budget-allocation multi-distribution MFN problem. To solve this novel and practical problem, the novel concept called the critical budget allocated vector is defined first. Without this new concept, it is impossible to solve this problem. The second novel concept called the maximal d-MP sets denoted by $P_{\max}$ is introduced and then a new algorithm is proposed based on the $P_{\max}$ to search for all d-MPs of the corresponding critical budget allocated vector. Note that the run time in searching for all d-MPs directly for all critical budget allocated vectors using the traditional algorithms without using the $P_{\max}$ is β times more than that of the proposed algorithm using the $P_{\max}$, where β is the number of all critical budget allocated vectors. Finally, the reliability of each critical budget allocated vectors is calculated in terms of the d-MPs obtained based on the $P_{\max}$ and the one with the best reliability is the optimal critical budget allocated vector we need.

The remainder of this paper is arranged as follows. Acronyms, notations, nomenclatures and assumptions are given in Section 2. Overviews of the network-based techniques in calculating the MFN reliability are presented in Section 3. Sections 4–6 provide the definitions, and discusses the properties of three major innovations in the proposed algorithm for the proposed BDMFN problem, reducing the computational complexity: the budget allocated vector in Section 4, the critical budget allocated vectors in Section 5, and the maximal set of d-MPs in Section 6. Section 7 presents the proposed algorithm for the BDMFN problem based on these three major innovations, with a discussion of the method's efficiency and effectiveness. Also, in Section 7, the proposed algorithm is demonstrated with a benchmark example to show how to solve the BDMFN problem. Concluding remarks are provided in Section 8.

2. Acronyms, notation, nomenclature and assumptions

Acronyms

MFN: the multi-state flow network
MP/MC: the minimal path/cut
d-MP/d-MC: the d-minimal path/cut

Notation

Pr(•): the probability of event • occurring
|•|: the number of elements or coordinates in •
n: the total number of nodes
m: the total number of arcs
β: the total number of critical budget allocated vectors
V: the set of nodes $V=\{1, 2, \ldots, n\}$
E: The set of arcs $E=\{e_1, e_2, \ldots, e_m\}$
$X\le(<)Y$: $X=(x_1, x_2, \ldots, x_\mu)\le(<)Y=(y_1, y_2, \ldots, y_\mu)$ if and only if $x_i\le(<)y_i$, $\forall\, i=1, 2, \ldots, \mu$.
$C_{\rm UB}$: the upper bound of the total budget that can be allocated to each arc
$G(V, E, W)$: a network (graph) with the node set V, the arc set E, and the set of the maximal arc W, the source node 1, and the sink node m.
C_i: the ordered set all possible budgets (sorted from smallest to largest) that can be allocated to $e_i\in E$, e.g., $C_1=\{0, 10, 20\}$ and $C_2=\{0, 5, 10, 15\}$ in Table 2.
$C_{i,j}$: the $j^{\rm th}$ element in C_i, e.g., $C_{1,1}=0$, $C_{1,2}=10$, and $C_{2,4}=15$ in Table 2.
b_i: the budget $b_i\in C_i$ that allocated to $e_i\in E$.
B : the budget vector $B=(b_1, b_2, \ldots, b_m)$. For example, $B=(b_1, b_2, \ldots, b_6)=(20, 15, 0, 20, 15, 0)$.
$S_i(b_i)$: the state vector of e_i under the budget b_i, e.g., $S_1(0)=(0, 1, 2)$ and $S_1(10)=S_1(20)=(0, 1, 2, 3)$ in Table 2. Note that $S_i(b_i)$ can be simplified to be $S(b_i)$ if we have known that b_i is the allocated budget for e_i.
$S(B)$: $S(B)=S((b_1, b_2, \ldots, b_m))=(S(b_1), S(b_2), \ldots, S(b_m))=(S_1(b_1), S_2(b_2), \ldots, S_m(b_m))$ is a vector of the state vector corresponding to the budget vector $B=(b_1, b_2, \ldots, b_m)$ and state vector $S(b_i)=S_i(b_i)$. For example, $S(B)=((0, 1, 2, 3), (0, 1, 2, 3, 4), (0, 1, 2, 3), (0, 1, 2, 3, 4), (0, 1, 2, 3, 4), (0, 1, 2))$, $S(b_1)=S_1(20)=(0, 1, 2, 3)$, and $S(b_2)=S_2(15)=(0, 1, 2, 3, 4)$ if $B=(b_1, b_2, \ldots, b_6)=(20, 15, 0, 20, 15, 0)$ from Table 2.
$R_{i,j}$: the probability vector of e_i under the budget $C_{i,j}$ such that the probability of the $k^{\rm th}$ coordinate in $S_{i,j}$ is the $k^{\rm th}$ coordinate in $R_{i,j}$, e.g., $R_{1,1}=(0.1, 0.3, 0.6)$, $R_{1,2}=(0.1, 0.2, 0.3, 0.4)$, and $R_{2,4}=(0.02, 0.18, 0.35, 0.40, 0.05)$ in Table 2
$S_{i,j}$: the $j^{\rm th}$ element in S_i, e.g., $S_{1,3}=S_1(20)=(0, 1, 2, 3)$ in Table 2.
D_i: $D_i=((S_{i,1}, R_{i,1}), (S_{i,2}, R_{i,2}), \ldots., (S_{i,\beta}, R_{i,\beta}))$ is the state distribution set of $e_i\in E$, which shows the capacities and the probabilities of each pre-given budget. For example, D_1 is all state distributions of e_1 and its values are listed under in e_1 of Table 2.
$D_{i,j}$: $D_{i,j}=(S_{i,j}, R_{i,j})$ is the $j^{\rm th}$ state distribution of e_i under budget $C_{i,j}$, e.g., $D_{1,1}=((0, 1, 2), (0.1, 0.3, 0.6))$, $D_{1,2}=((0, 1,$

Table 2
The budget-allocation multi-distribution in Fig. 1.

	arc	e_1			e_2				e_3		e_4		e_5				e_6		
	cost	0	10	20	0	5	10	15	0	10	0	20	0	5	10	15	0	5	10
	0	.10	.10	.05	.10	.05	.03	.02	.05	.03	.05	.03	.10	.05	.05	.03	.10	.10	.05
	1	.30	.20	.10	.40	.25	.20	.18	.25	.20	.20	.17	.40	.25	.20	.17	.30	.20	.10
state	2	.60	.30	.25	.50	.30	.37	.35	.30	.37	.35	.35	.50	.30	.35	.35	.60	.30	.25
	3		.40	.60		.40	.40	.40	.40	.40	.40	.40		.40	.40	.40		.40	.50
	4							.05				.05				.05			.10