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Aggregated combinatorial reliability model for non-repairable parallel phased-mission systems

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ABSTRACT

Phased-mission systems (PMSs) are common in many real-world applications. A PMS has to accomplish a mission with multiple phases with varied requirements on system operation and demand. Reliability evaluation of PMSs is more challenging than single-phased systems due to dynamics in system configuration (or structure function) and component behavior, as well as inherent inter-phase dependence. Though many efforts have been dedicated to the PMS reliability analysis, it is still difficult to evaluate the reliability of a large-scale PMS with many phases. In this paper, we make original contributions by proposing a new combinatorial model, named aggregated binary decision diagram (ABDD) for reliability analysis of non-repairable parallel PMSs subject to dynamic demand requirements. The proposed approach constructs a single ABDD model considering failure combinations in all phases simultaneously, enabling efficient analysis of PMSs with many phases. The approach is also extended to address the effects of fault level coverage. Examples of PMSs with different scales are analyzed to demonstrate application and efficiency of the proposed ABDD-based approach.

Notations

n	Number of components in the system
A_i	The ith component in the PMS, $i = 1, \dots, n$
M	Number of phases in the mission
T_j	Duration of phase j
$w_{i,j}$	Nominal capacity of A_i in phase j
d_j	Mission demand of phase j, $j = 1, \dots, M$
$F_i(\cdot), R_i(\cdot)$	Baseline cumulative distribution function/reliability function of A_i
$\alpha_{i,j}$	Lifetime acceleration factor for component A_i in phase j
Ξ_i	Mission phase in which component A_i fails
$p_{i,j}$	Probability that A_i fails in phase j, $p_{i,j} = \Pr\{\Xi_i = j\}$
$p_{i,M+1}$	Probability that A_i survives the mission, $p_{i,M+1} = \Pr\{\Xi_i = M+1\}$
$P_{i,j}$, $Q_{i,j}$	Probability that A_i fails/survives before $j+1$, $P_{i,j} + Q_{i,j} = 1$
$c_{i,j}$	Capacity of A_i in phase j, taking value of $w_{i,j}$ or 0
$\boldsymbol{c}_i$	Capacity vector of A_i, $\boldsymbol{c}_i = (c_{i,1}, \dots, c_{i,M})$
C_j	System capacity in phase j
$\boldsymbol{C}$	Capacity vector of the PMS, $\boldsymbol{C} = (C_1, \dots, C_M)$
e_l	A path in the ABDD
$\mathscr{J}_l$	Index of a critical phase for path e_l, $\mathscr{J}_l = \max_{1 \le j \le M} (C_{l,j} < d_j)$
$\mathscr{F}_l$	Set of failed components on a path e_l
$\mathscr{A}_l$	Set of failure combinations where all and only components in $\mathscr{F}_l$ fail in the mission
$\mathscr{B}_l$	Set of failure combinations where all and only components in $\mathscr{F}_l$ fail before phase $(\mathscr{J}_l + 1)$
$\mathscr{R}$	Set of failure combinations leading to mission success
E_l	Set of failure combinations aggregated in path e_l, $E_l = \mathscr{A}_l \cap \mathscr{R}$
β_r	Fault coverage probability for the rth component failure
R_S	System reliability with perfect fault coverage, $R_S = \Pr\{\mathscr{R}\}$
$R_{S,\,FLC}$	System reliability considering FLC

1. Introduction

A phased-mission system (PMS) is a system that has to accomplish a mission with multiple tasks sequentially [27]. These tasks have different requirements on the system configuration and operation, and the operating environments may vary during different phases. As a result,

Abbreviations: ABDD, Aggregated binary decision diagram; BDD, Binary decision diagram; FLC, Fault level coverage; IFC, Imperfect fault coverage; MDD, Multi-valued decision diagram; PMS, Phased-mission system

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the system would experience different stress levels, system success criteria and component failure behavior across the mission [29]. In addition, the state of one component at the end of one phase is identical to its state at the beginning of the next phase, which inherently introduces inter-phase dependence. Therefore, reliability modeling of PMSs is more challenging than single-phased systems.

Many efforts have been devoted to the reliability modeling of PMSs, see, e.g., [1,6,8,11,12,19,20,22,25,26,31]. In terms of adopted analytical modeling techniques for PMSs, there are state space oriented approaches based on Markov chains or Petri nets [5,11,23], combinatorial methods [14,20,24,25,30,31], and modular solutions based on binary decision diagram (BDD) and Markov chains [18]. Though the state space oriented approaches can explicitly model the state transition in the system and handle dynamic PMSs with random phase durations, they suffer the well-known space explosion problem for large-scale systems. In contrast, the combinatorial methods are effective in analyzing larger scale systems by exploiting BDD to reduce the computational complexity [9,10].

The BDD is an acyclic directed graph based on Shannon's decomposition of Boolean functions [4]. It has been widely used in reliability engineering since 1990s due to its computational advantage over traditional cut/path-sets based methods [13,21]. In 1999, BDD was first applied to the reliability modeling of PMSs [5], where the number of variables introduced to construct the system model is proportional to the number of system components multiplied by the number of phases. Phase algebra and new BDD generation and evaluation operations were developed to handle the dependence across phases. In Xing and Dugan [29], the BDD-based approach was extended to analyze reliability of a general PMS with combinatorial phase requirements, imperfect fault coverage (IFC) and multiple grade-level performance criteria. Xing [27] made a further extension to the PMS BDD method considering common-cause failures. However, due to the nature of the BDD model, all these existing BDD-based methods can still face severe computational complexity when the number of mission phases is large.

To address these difficulties, we propose a new combinatorial method named aggregated BDD (ABDD) for reliability modeling and analysis of parallel PMSs with heterogeneous components subject to dynamic demand. Real-world examples of such PMSs include power systems, engine systems of airplanes, and multi-processor data processing systems [12,15,28,29]. In the proposed approach, a single ABDD is constructed considering the requirements and success criteria of all the mission phases. The scale of ABDD is independent of the number of phases, which significantly reduces the computational complexity of the proposed approach.

The remainder of the paper is organized as follows. Section 2 gives a detailed description of the parallel PMS considered in this work. Section 3 discusses the traditional BDD-based method by constructing individual BDD for each phase. Section 4 presents the ABDD-based approach for system reliability evaluation. Section 5 gives examples of different scales to illustrate the application and efficiency of the proposed method. Section 6 concludes the paper and points out directions for future study.

2. Parallel PMS with heterogeneous components

Consider a non-repairable system with n statistically independent components A_1, ..., A_n working in parallel. Each component is binary, i.e., normal or failed. The lifetime of component A_i follows an arbitrary baseline distribution with cumulative distribution function $F_i(t)$. A failed component would stay in the failure state for the rest of the mission.

The system has to complete a mission with M successive phases. The duration of phase j has a predetermined length T_j, $j = 1, ..., M$. Component A_i may fail in any of the M phases or survive the mission. Denote the phase where A_i fails by Ξ_i, which is a discrete random variable and may take values of $1, ..., M+1$. Here, $\Xi_i = M+1$ indicates that A_i survives the mission. Due to variation of working conditions, the failure rate of a component may vary in different phases. Based the accelerated failure time model and the cumulative exposure model [17], component A_i in phase j suffers an acceleration factor $\alpha_{i,j}$. More specifically, the virtual lifetime of A_i in phase j in an interval Δt is $\alpha_{i,j}\Delta t$ when transformed to the baseline lifetime. Therefore, the probability that A_i fails in phase j is

$$p_{i,j} = \Pr\{\Xi_i = j\} = F_i\left(\sum_{k=1}^{j} \alpha_{i,k} T_k\right) - F_i\left(\sum_{k=1}^{j-1} \alpha_{i,k} T_k\right), \; j = 1, ..., M,$$

and the probability that A_i survives the mission is

$$p_{i,M+1} = \Pr\{\Xi_i = M+1\} = 1 - F_i\left(\sum_{k=1}^{M} \alpha_{i,k} T_k\right).$$

The probabilities that A_i fails before phase $j+1$ and A_i survives before phase $j+1$ are $P_{i,j} = \sum_{k=1}^{j} p_{i,j}$ and $Q_{i,j} = 1 - P_{i,j}$, respectively. Define $P_{i,0} = 0$ and $Q_{i,0} = 1$.

Each component has a nominal capacity $w_{i,j}$ in phase j when it is in the normal state. Here, $w_{i,j}$ can vary with j to account for the performance dependence of A_i on environments, working conditions, etc. Depending on the phase Ξ_i where A_i fails, the capacity $c_{i,j}$ that A_i can sustain in phase j can take value of $w_{i,j}$ or 0. Clearly, $c_{i,j}$ is a function of Ξ_i, $c_{i,j} = c_{i,j}(\Xi_i)$. The system capacity in phase j is equal to the sum of the working components' capacity: $C_j = \sum_{i=1}^{n} c_{i,j}$. The system capacity has to meet a predetermined mission demand d_j in phase j, and the mission succeeds if the demand is satisfied in all the phases. A practical example of such systems is the power system in a region, which consists of multiple power plants with variable capacity. The system capacity has to meet the power demand that may also vary with time.

Given the phase $\Xi_i = \xi_i$ that A_i fails for $i = 1, ..., n$, the capacity of each component and the system capacity in all the M phases are determined. Accordingly, the success or failure of the system, as a binomial random variable, is determined conditional on the failure combination $(\xi_1, ..., \xi_n)$. Define

$$\Omega = \{(\xi_1, ..., \xi_n) | 1 \le \xi_i \le M+1, \; 1 \le i \le n\}$$

and

$$\mathscr{R} = \left\{(\xi_1, ..., \xi_n) \middle| \sum_{i=1}^{n} c_{i,j}(\xi_i) \ge d_j \;\; \forall \, j = 1, ..., M\right\}.$$

Here, Ω is the universal set of failure combinations with $\Pr\{\Omega\} = 1$, and $\mathscr{R}$ denotes the set of failure combinations that lead to mission success. Thus, the system reliability is $R_S = \Pr\{\mathscr{R}\}$.

Peng, et al [20] developed a multi-valued decision diagram (MDD)-based approach to efficiently enumerate the failure combinations for system reliability calculation. However, the MDD method still has the worst-case computational complexity being exponential to the number of phases. In particular, the scale of the MDD model can increase rapidly with the increase of the number of phases. To facilitate the reliability evaluation of PMSs, a new ABDD-based approach is proposed to efficiently deal with missions involving many phases. In the following, we first present a preliminary approach using traditional BDDs, from which the ABDD is developed.

3. Preliminary approach based on BDD for each phase

Given that A_i is in operation at the beginning of phase j, it may fail or survive phase j. Correspondingly, there will be a capacity loss of $w_{i,j}$ to the system if A_i fails and 0 otherwise. The two possible scenarios can be modeled by a traditional BDD with two branches, as shown in Fig. 1. In the figure, the node $A_{i,j}$ denotes component A_i in phase j. Each branch represents a possible scenario of $A_{i,j}$ while the corresponding terminal value represents the capacity loss due to $A_{i,j}$.

We can build the BDD representation for each component in the