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Modeling of machine interference problem with unreliable repairman and standbys imperfect switchover

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ABSTRACT

This investigation is concerned with an M/G/1 machine interference problem with imperfect switchover of standbys, in which an unreliable repairman maintains a group of machines. An unreliable repairman means that the repairman is typically subject to unpredictable breakdowns. The time between two consecutive breakdowns follows an exponential distribution, and recovery time of the unreliable repairman follows a general distribution. The lifetime of operating/standby machines and the repair time of failed machines are exponentially and generally distributed, respectively. Using the method of supplementary variable, the stationary probability distribution is obtained. We develop some performance measures and system reliability indices. Furthermore, the cost effectiveness maximization is also discussed. Finally, a cost model is proposed to find the optimal numbers of operating and standby substations, which minimize the average cost per unit time.

1. Introduction

Machine interference problem has attracted much attention due to a wide range of real-world situations, such as computer systems, manufacturing/production assembly system systems, and aircraft maintenance. For more detail on this topic, the readers are referred to survey papers by Stecke and Aronson [23] and Haque and Armstrong [5]. In literature, machine interference problems with a reliable server/repairman have been investigated by several authors, including Gupta and Rao [4], Jain et al. [9], Ke and Lin [13], Wang et al. [24,27], and Yang and Chang [28]. But, in many real-world applications, we frequently encounter the case that the server/repairman breaks down unpredictably. The machine interference problems with server/repairman breakdowns can be referred to Jain and Bhargava [8], Yang and Chiang [29], Yen et al. [30], Jain and Meena [11], and references therein.

The standby redundancy is commonly used to improve reliability and availability of the system in reliability engineering. Liu et al. [21] studied a cold standby repairable system with working vacations and vacation interruption, where the lifetime of components and the repair time of the repairman follow Phase-type (PH) distributions. Zhang and Wang [31] proposed the extended geometric process repair model (EGPRM) for a cold standby repairable system with two dissimilar components and one repairman. Levitin et al. [18] recently proposed an iterative algorithm to evaluate the reliability of multi-state standby systems, where elements can be repaired in standby state, but cannot be repaired in operation. Lewis [20] was the first to introduce the concept of standby imperfect switchover in reliability study. Wang et al. [26] examined the reliability and availability characteristics for four various system configurations with warm standby components and imperfect switchover of standbys. Huang et al. [7] analyzed a repairable system with standby imperfect switchover in fuzzy environments. Reliability analysis associated with imperfect switchover and reboot delay of standbys in a repairable system with multi-repairmen conducted by Ke et al. [12]. As for a degradable system with standby imperfect switchover and reboot delay, El-Damcese and Shama [3] obtained the reliability function and mean time to system failure in explicit expressions. A matrix method was used to investigate the M/M/R machine interference problem with standby imperfect switchover and reboot delay by Hsu et al. [6], in which the coefficient of repair pressure is considered. For other related literature on the systems with standby imperfect switchover, we refer the readers to Wang and Chen [25], Jain and Rani [10], Levitin et al. [19], Ke and Liu [14], Lee [17], and Kuo and Ke [16]. Recently, Ke et al. [15] derived the stationary probabilities of an M/G/1 machine interference problem with imperfect switchover of standbys using the supplementary variable method. Shekhar et al. [22] provided the elaborate analysis of transient and stationary performance measures for the machine repair problem with geometric reneging and imperfect switchover of standbys.

In past, there has been extensive research on systems with imperfect switchover of standby. However, to the best of our knowledge, no study

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has considered an unreliable server/repairman in such systems. It motivates us to study the M/G/1 machine interference problem with server breakdowns and imperfect switchover of standbys. Consequently, this paper can be viewed as an extension of the model discussed by Ke et al. [15]. The paper is structured as follows. Section 2 describes the machine interference model with application to a smart grid. In Section 3, we use the supplementary variable method to analyze the probability distribution of the number of failed substations in the system. Some system characteristics and system reliability indices are developed in Section 4. We study the cost effectiveness maximization in Section 5. In Section 6, an average cost function is formulated to search for the optimum numbers of operating and standby substations at the minimum cost. In Section 7, we draw concluding remarks.

2. Model description

A smart grid is an electricity network that uses communication technology, sensors and other advance technologies for on-line monitoring of power transformers. It improves power reliability and enhances power utilization. The smart grid was introduced to overcome the weaknesses of conventional electrical grid with the aid of smart net meters (see [1]). Therefore, smart grid has attracted a huge attention in recent years. We consider a smart grid system to illustrate the potential application of the machine interference problem with an unreliable repairman and standbys imperfect switchover. There are $L = M + W$ homogeneous substations with M operating substations and W spare substations. Each substation fails independently of the others. The system consists of an imperfect switching mechanism when an operating substation fails. Thus, a failed substation is replaced by an available standby substation with probability $1 - q$. Assume that the lifetimes of the operating and standby substations follow exponential distributions with rates λ and α $(0 < \alpha < \lambda)$, respectively. Whenever an operating or standby substation fails, it is immediately sent to repair. The failed substation is repaired by a repairman. The time-to-repair of a failed substation has probability distribution function $G(t)(t \geq 0)$, probability density function $g(t)$, hazard rate function $\mu(t) = g(t)(1 - G(t))^{-1}$, and mean repair time $1/\mu$.

In addition, the repairman breaks down during busy periods, and the time between two consecutive breakdowns is assumed to be exponentially distributed with rate ξ. When a breakdown occurs, the repairman is sent to recovery. The recovery time has a general distribution $B(t)(t \geq 0)$, probability density function $b(v)$, hazard rate function $\beta(t) = b(t)(1 - B(t))^{-1}$, and mean recovery time $1/\beta$. When there are less than M substations in operation, the system shuts down. Note that throughout this paper for any function $\Omega(t)$, the notation $\bar{\Omega}(t)$ represents $1 - \Omega(t)$ and $\Omega^*(t)$ is the Laplace–Stieltjes transform of $\Omega(t)$.

The following notations are used throughout this paper.

M	Umber of operating substations
W	Number of standby substations
λ	Failure rate of operating substations
α	Failure rate of standby substations
ξ	Breakdown rate of the repairman
μ	Repair rate of failed substations
β	Recovery rate of the breakdown repairman
q	Unsuccessful witching probability

3. The analysis

We define the probabilities as follows:

$Q_{0,L}(t) \equiv$ probability that there are L substations in operation and the repairman is busy at time t;

$Q_{0,n}(t, y) \equiv$ probability that there are n substations in operation and the repairman is busy where the repair time of failed substations $Y(t)$ is between y and dy at time t, $L - 1 \leq n \leq M - 1$;

$Q_{1,n}(t, y, z) \equiv$ probability that there are n substations in operation and the repairman is breakdown where the repair time of failed substations $Y(t) = y$ and the recovery time of the breakdown repairman $Z(t)$ is between z and dz at time t, $L - 1 \leq n \leq M - 1$.

In steady-state, let us define

$$Q_{0,n}(y) = \lim_{t\to\infty} Q_{0,n}(t, y), \quad Q_{1,n}(y, z) = \lim_{t\to\infty} Q_{1,n}(t, y, z),$$

$$Q_{0,n} = \int_0^\infty Q_{0,n}(y)dy \quad \text{and} \quad Q_{1,n} = \int_0^\infty \int_0^\infty Q_{1,n}(y, z)dydz.$$

The state-transition-rate diagram of the system is given in Fig. 1. We relate the states of the system at two consecutive time epochs, t and $t + dt$, and take $dt \to 0$. The integro-differential equations governing the system are constructed as following:

$$\gamma_0 Q_{0,L} = \int_0^\infty \mu(y)Q_{0,L-1}(y)dy, \tag{1}$$

$$\frac{dQ_{0,L-1}(y)}{dy} + \left(\gamma_1 + \xi + \mu(y)\right)Q_{0,L-1}(y) = \int_0^\infty \beta(z)Q_{1,L-1}(y, z)dz, \tag{2}$$

$$\begin{aligned}\frac{dQ_{0,L-i}(y)}{dy} &+ \left(\gamma_i + \xi + \mu(y)\right)Q_{0,L-i}(y)\\ &= \lambda_{i-1}Q_{0,L-i+1}(y) + \sum_{j=1}^{i-2}\varphi_{i-j-1}Q_{0,L-j}(y)\\ &+ \int_0^\infty \beta(z)Q_{1,L-i}(y, z)dz, 2 \leq i \leq W,\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{dQ_{0,M-1}(y)}{dy} + (\xi + \mu(y))Q_{0,M-1}(y) &= \gamma_W Q_{0,M}(y) + \sum_{j=1}^{W-1}\theta_j Q_{0,L-j}(y)\\ &+ \int_0^\infty \beta(z)Q_{1,M-1}(y, z)dz,\end{aligned} \tag{4}$$

$$\frac{dQ_{1,L-1}(y, z)}{dz} + \left[\gamma_1 + \beta(z)\right]Q_{1,L-1}(y, z) = 0, \tag{5}$$

$$\begin{aligned}\frac{dQ_{1,L-i}(y, z)}{dz} &+ \left[\gamma_i + \beta(z)\right]Q_{1,L-i}(y, z)\\ &= \lambda_{i-1}Q_{1,L-i+1}(y, z) + \sum_{j=1}^{i-2}\varphi_{i-j-1}Q_{1,L-j}(y, z), \; 2 \leq i \leq W,\end{aligned} \tag{6}$$

$$\frac{dQ_{1,M-1}(y, z)}{dz} + \beta(z)Q_{1,M-1}(y, z) = \gamma_W Q_{1,M}(y, z) + \sum_{j=1}^{W-1}\theta_j Q_{1,L-j}(y, z), \tag{7}$$

where $\gamma_i = M\lambda + (W - i)\alpha, 0 \leq i \leq W$; $\lambda_i = M\lambda(1 - q) + (W - i)\alpha, 0 \leq i \leq W - 1$; $\varphi_i = M\lambda q^i(1 - q), 1 \leq i \leq W - 1$; $\theta_i = M\lambda q^{W-i}$, $0 \leq i \leq W - 1$.

The boundary conditions are given as follows:

$$Q_{0,L-1}(0) = \int_0^\infty \mu(y)Q_{0,L-2}(y)dy + \lambda_0 Q_{0,L}, \tag{8}$$

$$Q_{0,L-i}(0) = \int_0^\infty \mu(y)Q_{0,L-i-1}(y)dy + \varphi_{i-1}Q_{0,L}, \quad 2 \leq i \leq W, \tag{9}$$

$$Q_{1,L-i}(y, 0) = \xi Q_{0,L-i}(y), \quad 1 \leq i \leq W + 1. \tag{10}$$

Solving the above integro-differential Eqs. (5)–(7) with boundary conditions (8)–(10), we obtain

$$Q_{1,L-1}(y, z) = \xi\bar{B}(z)e^{-\gamma_1 z}Q_{0,L-1}(y), \tag{11}$$