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Failure-mode importance measures in structural system with multiple failure modes and its estimation using copula

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A R T I C L E I N F O

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A B S T R A C T

In the structural reliability analysis, there exist multiple failure modes. Influence of each failure mode on the structural system reliability needs to be considered, because it is significant for simplifying the system model and improving the performance of the system. By using the concepts of the importance measures(IM) in probability risk assessment (PRA), the importance indices in PRA are extended to measure the failure mode contribution to the structural system reliability, and the analytical solutions of the failure mode importance measure for the parallel and series structural systems are derived firstly. Then, copula method is proposed to estimate the importance indices for the nested hybrid structural systems. At last, one numerical example with a parallel structure and two engineering examples are employed to analyze the failure mode importance and to test the two estimates based on the analytical solution and the copula method. Result indicates that five PRA importance indices can well reflect the failure mode importance. It also shows that the copula method is highly suitable for computing PRA IMs in the intricate structural systems.

1. Introduction

It is common for structural system in reliability engineering that there are multiple failure modes, such as strength failure, stiffness failure, fatigue failure and stability failure, etc., because of different requirements. Similar to the components-system, the multiple failure modes that structural system is subject to also have parallel, series and hybrid relationship. According to influence of failure modes on system reliability, designers or decision makers can identify which modes must be considered in reliability analysis and the ones with little importance to system can be ignored, and it makes the reliability analysis more rational and efficient. Thus, a need to quantify the failure modes importance has arisen for the reliability analysis of structural system with multiple failure modes. By using of the importance ranking of each failure modes, engineers can improve the system performance by designing or changing the important failure modes; for less important failure modes, it is reasonable to remove those failure modes to reduce the complication of the system, simplify the analysis and improve the computational efficiency.

Because the failure modes may be caused by same random input variables more or less in engineering, they are correlated, which makes the failure modes importance analysis more complicated. Usually, measuring the failure modes importance is just based on the failure probability of the single failure mode, it is not comprehensive. Thus, it is imperative to build the importance measures for multiple failure modes. In fact, similar but limited work has been done by some researches [1–4]. Ref. [1] put forward two schemes to measure the importance of the failure modes. Ref. [2] proposed a surrogate-based approach to identify and weigh the importance of the failure modes. Ref. [3] estimated the relative contributions of each failure mode to the system failure probability. In the present literature, lots of works focus on the importance measure analysis of the input variables on the statistics characteristics of the outputs [5–7], few works about the importance measures (IM) of the failure modes with correlated effect on the structural system can be found.

In this article, two aspects of the failure mode IM are concerned: the interpretation of the failure mode IMs using probability risking assessment (PRA) measures, and the estimation method for the failure mode IMs using copula.

First, from the concepts of the previous works about PRA IMs in system reliability analysis [8–15], it is well known that the PRA IMs are very useful for identifying the importance of the components or the basic events contributed to the system. In the past several years, many PRA IM indices have been developed for measuring the importance of the components or the basic events to the system reliability, such as Birnbaum, Bayes, Fussell-Vesely, Criticality importance, Risk achievement/reduction worth, etc. And each PRA IM implies different aspects

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Table 1
List of the generally used PRA IMs.

IMS	Index	Definition
Birnbaum (bm)	$I(bm)_i$	$I(bm)_i = P(S^-\|X_i^-) - P(S^-\|X_i^+)$
Bayes (Bay)	$I(bay)_i$	$I(bay)_i = P(X_i^-\|S^-)$
Criticality (cm)	$I(cm)_i$	$I(cm)_i = [P(S^-\|X_i^-) - P(S^-\|X_i^+)] \times \frac{P(X_i^-)}{P(S^-)}$
Risk Achievement Worth (raw)	$I(raw)_i$	$I(raw)_i = \frac{P(S^-\|X_i^-)}{P(S^-)}$
Risk Reduction Worth (rrw)	$I(rrw)_i$	$I(rrw)_i = \frac{P(S^-)}{P(S^-\|X_i^+)}$

of importance. In this contribution, PRA IMs are extended to measure the influence of each failure mode on the structural system failure probability.

Second, the estimation of PRA IM based on copula method is developed. In simple structural system, such as series or parallel failure modes system, the logical relationship between the mode failure and the system failure can be obtained by the analytical method. However, due to complicated relationship between the failure modes and the structural system with multiple nested hybrid failure modes, the analytical solution is unavailable, and a general solution must be developed for this case. It is well known that copula is a very useful tool for understanding the relationship between the random variables [16–21]. Based on this character of the copula function, the PRA IM indices of the failure modes in the structural system is expressed as the copula function, and the empirical copula function is employed to estimate the copula function. And the method only needs one set of single sample for estimation all the PRA IM indices for the structural system.

The rest of article is organized as follows. Section 2 reviews the definitions of PRA IMs in the component system. Section 3 extends PRA IMs to measure the failure mode importance to the structural system. The connotation of PRA IMs for the structural system with multiple failure modes is explained, and the analytical solutions are derived for the series and parallel system respectively. Section 4 proposes the copula based method and its estimation procedure. Section 5 presents a numerical example, i.e. a parallel system with multiple failure modes. Two methods, analytical solution and copula, are used to estimate the PRA IM indices, and the comparison of the results estimated by the two methods demonstrate the accuracy of the proposed copula based method. Section 6 applies the PRA IMs to deal with two engineering examples. Section 7 gives some conclusions.

2. Review of PRA importance measures

In Table 1 several typical PRA IMs and their definitions are given [8–15], where S^-stands for the system is disabled, X_i^- means the ith component is failed, and S^+ or X_i^+ are opposite respectively. For the coherent system consisted of n components, PRA IM indices indicate the influence of the ith component on the failure probability of the system.

Birnbaum IM is defined to measure the effect of the ith component on the system failure probability. $I(bm)_i$ is the difference between the conditional failure probabilities of the system when the ith component is disabled and when it does not fail.

Bayes IM is defined to identify which component causes the system failure. $I(bay)_i$ measures the probability that the ith component fails, given that the system fails.

Criticality IM has the similar meaning as Birnbaum IM, furthermore, it enables to discriminate components that have the same Birbaum IM by its own failure probability.

The risk achievement worth (RAW) IM and risk reduction worth (RRW) IM are both defined as a ratio. RAW IM is a ratio of the conditional system failure probability with the ith component failure to the unconditional system unreliability, which presents a measure of the 'worth' of the ith component in 'achieving' the present level of risk, and reflects the maintained importance for the system. RRW IM is defined as the ratio of the system unreliability to the conditional system unreliability if the ith component is replaced by a prefect component, which represents the maximum decrease of the risk that may be expected by increasing the reliability of the ith component.

The estimation methods of PRA IMs mentioned above are often solved through fault tree/event tree or binary decision diagrams in the component-system problem. However, one of the prerequisites is that each component or event is independent. Thus, for correlated components or events with each other in system, the research works are limited [22].

3. Connotation and interpretation for the failure mode IM extended from PRA IM

Consider multiple failure modes with the corresponding performance functions $Y_j = g_j(\boldsymbol{X})$ $(j = 1, 2, ...m)$, where $\boldsymbol{X} = (X_1, X_2, ..., X_n)$ is the n-dimensional input vector of these performance functions. The joint probability density function (PDF) of $\boldsymbol{X}$ is denoted as $f_{\boldsymbol{X}}(\boldsymbol{x})$. For reliability analysis with a single failure mode, the failure probability can be formulated as $P_f = P(Y \le 0) = \int_{Y\le 0} f_{\boldsymbol{X}}(\boldsymbol{x})d\boldsymbol{x}$. For a system with multiple failure modes, the failure probability of a series system is defined as $P_f = P\{Y_1 \le 0 \cup Y_2 \le 0 \cup ... \cup Y_m \le 0\}$, while the failure probability of a parallel system is $P_f = P\{Y_1 \le 0 \cap Y_2 \le 0 \cap ... \cap Y_m \le 0\}$

Based on the definition above, the PRA IM indices can be extended to measure the effects of the failure mode importance on the structural system with multiple failure modes, and the corresponding explanations can be presented as follows.

Birnbaum IM for the jth failure mode is defined as the difference between two conditional failure probabilities of the structural system, and one condition is the jth failure mode happens and other failure models does not happen. So $I(bm)_i$ measures the change of the structural system failure probability when the jth mode is failed or not.

Let Y_j and S denote the limit state functions of the jth failure mode and the system respectively.$S^- = \{S \le 0\}$ and $Y_j^- = \{Y_j \le 0\}$ represent that the system is failed and the jth mode is failed respectively. For the series system, since $\{S \le 0\} = \{\bigcup_{j=1}^{m} Y_j \le 0\}$ (m is the total number of the failure modes) in the series system, $P(S \le 0, Y_j \le 0) = P(Y_j \le 0)$ holds. For the parallel system, $\{S \le 0\} = \{\bigcap_{j=1}^{m} Y_j \le 0\}$, so $P(S \le 0, Y_i \le 0) = P(S \le 0)$ can be obtained similarly. Thus, in the series system, Birnbaum IM of the jth failure mode can be derived as:

$$I(bm)_j = P(S^-\left|Y_j^-\right.) - P(S^-\left|Y_j^+\right.) = \frac{P(S \le 0, Y_j \le 0)}{P(Y_j \le 0)} - \frac{P(S \le 0) - P(S \le 0, Y_j \le 0)}{1 - P(Y_j \le 0)} = \frac{1 - P(S \le 0)}{1 - P(Y_j \le 0)} = \frac{P(S > 0)}{P(Y_j > 0)} \quad (1)$$

In the parallel system, Birnbaum IM for the jth failure mode can be similarly derived as:

$$I(bm)_j = \frac{P(S \le 0)}{P(Y_j \le 0)} \quad (2)$$

Bayes IM of failure mode can be used to identify which failure mode causes the system failure. $I(bay)_i$ measures how much the probability of