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# Reliability evaluation of multi-state series systems with performance sharing

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ABSTRACT

In this paper, a new reliability model for a multi-state system (MSS) with performance sharing is proposed. The MSS consists of $N$ multi-state units connected in series. Each unit in the system has a random performance level and a random demand. If the performance of a unit exceeds its demand, the surplus performance can be transmitted to its adjacent units through intermediate transmitters. Each transmitter has a random capacity, through which only a limited amount of performance can be transmitted. The entire system fails if the demand of any unit is not satisfied. An algorithm based on the universal generating function (UGF) is developed to evaluate the reliability of the system. Analytical and numerical examples are provided to validate the proposed method. Examples show that the developed algorithm is efficient in system reliability evaluation.



## 1. Introduction

### 1.1. Background and motivation

Classical reliability theory is devoted to binary-state reliability models in which a system or a component has only two possible states, i.e., perfect working and complete failure. However, many real-world systems have multiple performance levels in operating. Such systems are called multi-state systems (MSSs) [1]. For example, the performance of a power system is characterized by its generating capacity with multiple levels, and the performance of a distributed computer system is measured by its speed of data processing, which is also a multiple-value variable.

Performance sharing mechanism appears in some practical complex systems. The performance of each unit should meet its individual demand first, and the surplus performance can be transmitted to other units that experience performance deficiency. Performance sharing phenomena widely exists in power supply systems, production systems, and computer systems [2]. A typical model for a MSS with performance sharing was proposed by Levitin [3]. All the units in the MSS were connected by a common bus, through which the surplus performance of a unit can be shared by all the other units.

Existing research assumes that the MSS with performance sharing works with a common bus, by which the total surplus performance is redistributed over the whole system. However, units in a practical system are often connected with a certain configuration, and the surplus performance of a unit can only be shared by units directly connected to it. For example, the surplus power of a power plant is firstly shared by its adjacent power plants, and then extra power can be further shared by other power plants through the connections between them. A system fails if any individual demand in the system is not satisfied. The aim of this paper is to evaluate the reliability of such multi-unit MSS with performance sharing between adjacent units.

### 1.2. Related work

The reliability evaluation and optimization of MSSs have received much attention [4–11]. In Ref. [9], the maintenance optimization of a multi-state series-parallel system considering economic dependence and state-dependent inspection intervals was addressed. Peng et al. [11] discussed the reliability analysis and optimal design of series-parallel phased-mission systems subject to fault-level coverage. A multi-state k-out-of-n system was proposed by Huang et al. [12]. The system is in state $j$ if and only if at least $k_j$ components are in a state better than state $j$. In Refs. [13,14], some efficient algorithms were presented to evaluate the system reliability for the multi-state k-out-of-n system. More extensions can also be found from Refs. [15,16].

Performance sharing systems were firstly studied by Lisnainski and Ding [2], in which a system with two multi-state units was considered (i.e.,a main unit and a reserve unit). The surplus performance transmission in Ref. [2] was monodirectional, i.e., from the reserve unit to the main unit. As an extension, Levitin [3] proposed a multi-unit sys-

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tem with performance sharing through a common bus. All the units in the system were connected by the common bus, through which the surplus performance of some units can be shared by all the other units. Yu et al. [17] evaluated the instantaneous availability of a multi-unit system with common bus performance sharing. Xiao and Peng [18] discussed the optimal allocation and maintenance of multi-state elements in series-parallel systems with common bus performance sharing. Xiao et al. [19] evaluated the availability of systems by considering the effect of loading and protection of external impacts on multi-state systems with performance sharing mechanism. Zhai et al. [20] researched the defense and attack strategies for a system with a common bus performance-sharing and under intentional attacks. In Refs. [21,22], the multi-state units in a complex system were divided into several performance sharing groups that were connected in series. Algorithms were presented to evaluate the reliability of the systems.

### 1.3. Contribution of this work

Although common bus performance sharing can describe a class of engineering systems in reality, it is not suitable in modeling a performance sharing system with a specific connection structure. This paper addresses the reliability evaluation problem of such performance sharing systems. In this research, the surplus performance of each unit is firstly transmitted to the units adjacent to it and then further shared by other non-adjacent units. The amount of performance that can be transmitted is limited by the capacity of the transmitter between units. The essential difference between the proposed model and existing performance sharing models is that the existing models assume that the performance of the units in a system can be shared through a single general common bus, and the shared performance between any two units is independent from the distance between the two units. On the contrary, the considered model assumes that the performance of two units is shared through the transmitter between them. If two units are unconnected, they cannot share their surplus performance directly. Therefore, the shared performance depends on the distance between the two units. Two units that have larger distance are more difficult to share performance.

Due to the complexity of the performance sharing problem studied, this paper only discuss the reliability evaluation of a multi-state series system. Based on the universal generating function (UGF) technique [23], an efficient algorithm is developed to compute the reliability of the system. The UGF technique is a useful tool to analyze MSSs [24–27]. Reliability of systems with general structures, such as series-parallel structure or parallel-series structure, can also be computed using the algorithm proposed in this paper. The developed method provides references for evaluating reliability of more complex systems.

### 1.4. Overview

The paper is organized as follow. Section 2 introduces a performance sharing model of the multi-state series system. The developed model is analyzed in Section 3, and the UGF-based algorithm for evaluating the reliability of the performance sharing system is presented in Section 4. Section 5 gives analytical and numerical examples. Conclusions are made in Section 6.

## 2. Notations and model description

### 2.1. Notations

| | |
|---|---|
| $N$ | Number of units in the series system |
| $k_i$ | Number of performance levels of unit $i$ |
| $\mathbf{g_i}$ | Performance level set $\{g_{ik_i}, \cdots, g_{i1}\}$ of unit $i$ |
| $G_i(t)$ | Random performance level of unit $i$ at time $t$ |
| $h_i$ | Number of demand levels of unit $i$ |
| $\mathbf{w_i}$ | Demand level set $\{w_{ih_i}, \cdots, w_{i1}\}$ of unit $i$ |
| $W_i(t)$ | Random demand level of unit $i$ at time $t$ |
| $G_i$ | Random performance level of unit $i$ in the stationary state |
| $W_i$ | Random demand level of unit $i$ in the stationary state |
| $m_i$ | Number of capacity levels of the transmitter between units $i$ and $i+1$ |
| $\mathbf{c_{i,i+1}}$ | The set of capacity levels |
| $C_{i,i+1}$ | Random capacity level of the transmitter between units $i$ and $i+1$ |
| $B_k$ | Subsystem composed of units $1, \cdots, k$ |
| $Z_i$ | Difference between the performance and the demand of unit $i$, i.e.,$G_i - W_i$ |
| $S_k$ | Cumulative surplus performance of $B_k$ |
| $\varphi(\cdot,\cdot)$ | A bivariate function |
| $Y_k$ | $\varphi(S_k, C_{k,k+1})$ |
| $U_{G_i}(z)$ | UGF of the performance $G_i$ |
| $U_{W_i}(z)$ | UGF of the demand $W_i$ |
| $U_{Z_i}(z)$ | UGF of $Z_i$ |
| $U_{S_k}(z)$ | UGF of $S_k$ |
| $\eta_k(z)$ | UGF of $C_{k,k+1}$ |
| $U_{Y_k}(z)$ | UGF of $Y_k$ |
| $\alpha_{ij}$ | Probability that the performance $G_i$ takes value $g_{ij}$ |
| $\beta_{ij}$ | Probability that the demand $W_i$ takes value $w_{ij}$ |
| $\gamma_{ij}$ | Probability that the state $Z_i$ takes value $z_{ij}$ |
| $p_{kj}$ | Probability that the cumulative performance $S_k$ takes value $s_{kj}$ |
| $q_{kl}$ | Probability that the transmission capacity $C_{k,k+1}$ takes value $c^l_{k,k+1}$ |
| $\theta_{kj}$ | Probability that the state $Y_k$ takes value $y_{kj}$ |
| $R$ | Reliability of the system |

### 2.2. Model description

The series system investigated consists of $N$ multi-state units, and there are performance transmitters between adjacent units. Unit $i$ has $k_i$ different performance levels, which are represented by the set $\mathbf{g_i} = \{g_{ik_i}, \cdots, g_{i1}\}$ ($g_{ij} > g_{il}$ for $j > l$). The performance level $G_i(t)$ of unit $i$ at time instant $t$ is a random variable taking values from $\mathbf{g_i}$. The performance level $\{G_i(t), t > 0\}$ forms a discrete-state continuous-time stochastic process (e.g., semi-Markov process). There is a demand $W_i(t)$ for unit $i$ at time $t$. The set $\mathbf{w_i} = \{w_{ih_i}, \cdots, w_{i1}\}$ stands for the set of possible demand levels corresponding to unit $i$, where $h_i$ is the number of different demand levels. The demand $\{W_i(t), t > 0\}$ also follows a discrete-state continuous-time stochastic process.

The stationary performance of a system is usually of more interest if a system is operating in a rather long time period. When all the involved stochastic processes have stationary distributions, the reliability of the multi-state series system can be calculated.

The diagram of a N-unit multi-state series system with performance sharing is shown in Fig. 1. Let $G_i$ and $W_i$ be respectively the random performance and demand of unit $i$ in the stationary state. The variable $Z_i = G_i - W_i$ represents the difference between performance production and demand of unit $i$. The surplus performance of unit $i$ is $Z_i$ if $Z_i > 0$. On the other hand, the performance deficiency of unit $i$ is $-Z_i$ if $Z_i < 0$. Units $i$ and $i+1$ can share their surplus performance with each other