Reliability and maintenance policies for a two-stage shock model with self-healing mechanism

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ABSTRACT

In this paper, a two-stage shock model with self-healing mechanism is proposed as an extension of cumulative shock and delta-shock models. A change point is introduced to describe the two-stage failure process of a system and defined as the moment when the cumulative number of valid shocks reaches d. Before the change point, the system can heal the damage caused by a valid shock when the number of delta-invalid shocks reaches k in the trailing run of invalid shocks. Equivalently, the damage caused by previous i valid shocks can be healed when the number of delta-invalid shocks falls in $[ik, (i + 1)k)$ in the run of invalid shocks. The system loses self-healing ability when it reaches the change point, and then fails when the cumulative number of valid shocks reaches a prefixed value n $(n > d)$. Based on the established model, the finite Markov chain imbedding approach is employed to obtain the probability mass function, the distribution function and the mean of shock length. Three preventive maintenance policies are proposed for the system under different monitoring conditions, and corresponding optimization models are constructed to determine the optimal quantities. Finally, numerical examples are given for the proposed model.

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1. Introduction

In general, engineering systems are subject to the internal degradations and external shocks, which may result in system failure [1–4]. Shock models are mainly used to study the external causes, such as overload, vibration and so on, which have received lots of concerns in applied probability, operational research and engineering reliability [5,6]. In a shock model, a system (or component) is subject to shocks occurring randomly in time. Over the past several decades, various shock models have been defined and studied. Basically, those shock models fall into five categories [7]: Cumulative shock model [8], extreme shock model [9,10], run shock model [11], δ-shock model (delta-shock model) [12] and mixed shock model [13,14]. For cumulative shock model, the system failure results from the cumulative effect of shocks, while for extreme shock model, the system breaks down as soon as the magnitude of an individual shock is above a critical threshold. The system works until k consecutive shocks occur with their magnitudes exceeding a critical level in run shock model. In terms of δ-shock model, the system fails when the time lag between two adjacent shocks is less than a given critical value. In addition, a mixed shock model can be constructed by combining two different shock models. For example, the system fails as soon as the cumulative magnitude of shocks exceeds a critical level or

k consecutive critical shocks occur, whichever comes first. This is the combination of the cumulative and run shock models [14].

The previous studies on shock models are devoted to two aspects: reliability analysis and maintenance policy optimization. On the one hand, in the domain of reliability analysis, reliability indices, such as system reliability, mean residual lifetime, survival and failure rate functions, have been widely discussed. Esary et al. [15] analyzed the life distribution of a device under cumulative shock model in the case that shocks' arrival follows a homogenous Poisson process. As an extension of their work, Hameed and Proschan [16] considered the case where shocks occur following a non-homogenous Poisson process. Thall [17] derived the survival function when shocks occur according to a homogeneous Poisson cluster process. Neuts and Bhattacharjee [18] discussed the distribution of the time to failure when the interarrival times between two adjacent shocks follow a phase-type distribution. Mallor and Santos [19] studied a general reliability shock model and provided the distribution function of the failure time and its mean value. Li and Zhao [12] first obtained a general lifetime distribution for a general complex system in the δ-shock model. Eryilmaz [7] explored three different discrete time shock models involving runs and obtained the survival functions of the systems. Furthermore, some recent contributions involving reliability analysis for shock models have been studied in the litera-

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ture [20–27]. On the other hand, maintenance policies, such as age-based and condition-based maintenance policies, have been applied to shock models in order to reduce the losses of random failures. Nakagawa [28] introduced a replacement policy in which an item is replaced when the total damage exceeds a threshold. Qian et al. [29] considered that the system undergoes a minimal repair when the total damage exceeds a critical level and a replacement at time T or at failure, whichever occurs first. Nakagawa [1] gave a detailed introduction to shock and damage models and kinds of optimum maintenance policies were discussed theoretically. Eryilmaz [5] employed an age-based maintenance policy to minimize the expected cost per unit time by determining the optimal replacement time. Lam and Zhang [30], Wang and Zhang [31], Montoro-Cazorla and Pérez-Ocón [32] adopted the replacement policy N based on the number of failure of the system. Besides, some maintenance policies for systems suffering from internal deterioration and sudden shocks were discussed in [33-37].

Recently, some studies on shock models focused on the healing ability of systems, which includes 'curable' shock process and self-healing mechanism. Cha and Finkelstein [38] as well as Finkelstein and Cha [39], introduced the 'curable' shock process in which that each shock coming from a nonhomogeneous Poisson process can trigger a failure of a system not immediately, but with delay of some random time. Each shock can be cured (repaired) with certain probabilities in the delayed time as if it did not happen. Similarly, many systems and products are able to annihilate the damages due to their intrinsic resistance to external damages, which is called the self-healing effect. Liu et al. [24] and Cui et al. [40] discussed the system reliability considering the self-healing effect after each random shock, in which self-healing effect follows a specific effect function.

For most practical systems, they will lose the healing ability when the damage reaches a threshold. However, the abovementioned shock models did not consider the limit of the healing ability under the cumulative effect of the shocks. In this paper, the change point is adopted to describe the limit of healing ability. Moreover, we construct a new self-healing mechanism, which can be activated and the damage is healed immediately when the system meets a certain condition.

Based on the change point and self-healing mechanism, a two-stage shock model with self-healing mechanism is proposed. In this new model, a shock that results in a certain degree of system damage is called a valid shock, otherwise it is called an invalid shock. The δ-invalid shock denotes an invalid shock whose time lag with the preceding shock exceeds a given threshold δ. A change point is introduced to divide the failure process into two stages. The stage before the change point is defined as stage 1, in which δ-invalid shocks can trigger the self-healing mechanism. When the system meets the condition of change point, it switches to stage 2 and loses the self-healing ability. In addition, three preventive maintenance policies are proposed for the system corresponding to different monitoring conditions.

The proposed shock model with self-healing mechanism can be widely applied in engineering field. For example, one kind of self-healing materials, developed by material scientists, is able to identify the occurrence of damage and repair itself under some appropriate stimuli. This self-healing material has been widely used in polymer matrix composites products [41]. In the following, a more specific example in aviation industry will be presented. An aircraft using this self-healing material may encounter some turbulences (valid shocks) which cause some damages to the airframe. When the cumulative damage exceeds a critical level, the aircraft will break down and incur enormous losses. However, if no valid shock occurs during a period of time and the airframe is subject to some appropriate stimuli (δ-invalid shocks), the self-healing mechanism can be triggered and the damage caused by the latest valid shock can be healed. Such stimuli could be heat, light, electrical fields or moisture [41], which can be regarded as δ-invalid shocks. However, when the cumulative damage reaches a certain amount, the aircraft will lose the self-healing ability and then the damage caused by valid shocks starts to accumulate. Obviously, the reliability of air-

craft can be increased and its service life can be extended effectively by applying the self-healing materials.

The remainder of the paper is organized as follows. In Section 2, the proposed shock model with assumptions is described in detail. The probabilistic analysis of the system under this model is presented in Section 3. In Section 4, three preventive maintenance policies are proposed and corresponding optimization models are constructed to determine the optimal quantities. In Section 5, numerical examples are presented for the proposed model. Finally, conclusions of this paper and future possible continuations are given in Section 6.

Notations

X_i	time lag between the $(i-1)$-th and i-th shocks, $i = 1, 2, 3, \ldots$
N_1	total number of shocks until the change point occurs
N_2	total number of shocks until the system fails
T_1	Sojourn time in stage 1
T_2	lifetime of the system
n	cumulative number of valid shocks that cause the system failure
d	cumulative number of valid shocks that make the system come to the change point
k	minimum number of δ-invalid shocks that can heal the damage caused by a single valid shock
Ω	state space of the Markov chain $\{Y_m, m \geq 0\}$
Λ	one-step transition probability matrix of the Markov chain $\{Y_m, m \geq 0\}$
Φ	one-step transition probability matrix of the Markov chain $\{Y_m, m \geq 0\}$ in stage 1
γ	order of the matrix Λ
γ_1	order of the matrix Φ
c_{p1}	cost of replacing a non-failed system in stage 1
c_{p2}	cost of replacing a non-failed system in stage 2
c_f	cost of replacing a failed system

2. Model assumptions and model description

2.1. Model assumptions

We introduce the shock model with self-healing mechanism by making the following assumptions.

1) The system is operating in random environment, which is subject to a sequence of randomly occurring shocks over time.
2) Each shock has two possible types: Valid shock with the probability of p, and invalid shock with the probability of q (i.e., $p + q = 1$). A valid shock causes a certain magnitude of damage to the system, while an invalid shock has no damage on the system.
3) Let X_i denote the time lag between the $(i-1)$-th and i-th shock, $i \geq 1$. Assume that $X_1, X_2, \ldots$ are independent and identically distributed (i.i.d.) random variables following an exponential distribution with parameter λ, which can be expressed as $X_i \sim Exp(\lambda)$.
4) X_i is independent of the type of each shock.
5) A self-healing behavior might be triggered by invalid shocks. However, the system will lose the self-healing ability when the cumulative number of valid shocks exceeds a critical level.

2.2. Model description

Based on the general assumptions of the model, shocks arrive following a Poisson process. If a valid shock occurs and is followed by some invalid shocks, the damage resulting from previous valid shocks might be healed. The δ-invalid shock denotes an invalid shock whose time lag with the preceding shock exceeds a given threshold δ. The effect of self-healing can be described as that it can heal the damage caused by previous i valid shocks when the number of δ-invalid shocks falls in $[ik, (i+1)k)$ in the trailing run of invalid shocks. Specifically,