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Bayesian step stress accelerated degradation testing design: A multi-objective Pareto-optimal approach

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A B S T R A C T

Step-stress accelerated degradation testing (SSADT) aims to access the reliability of products in a short time. Bayesian optimal design provides an effective alternative to capture parameters uncertainty, which has been widely employed in SSADT design by optimizing specified utility objective. However, there exist several utility objectives in Bayesian SSADT design; for the engineers, it causes much difficulty to choose the right utility specification with the budget consideration. In this study the problem is formulated as a multi-objective model motivated by the concept of Pareto optimization, which involves three objectives of maximizing the Kullback-Leibler (KL) divergence, minimizing the quadratic loss function of *p*-quantile lifetime at usage condition, and minimizing the test cost, simultaneously, in which the product degradation path is described by an inverse Gaussian (IG) process. The formulated programming is solved by NSGA-II to generate the Pareto of optimal solutions, which are further optimally reduced to gain a pruned Pareto set by data envelopment analysis (DEA) for engineering practice. The effectiveness of the proposed methodologies and solution method are experimentally illustrated by electrical connector's SSADT.

1. Introduction

Modern products and engineering systems are known for their high reliability and long service life and it is hard to collect time-to-failure data under typical operating conditions. To extrapolate the reliability and lifetime within a limited period, accelerated degradation testing (ADT), as an important activity in reliability engineering, has been widely used to obtain the degradation data of a product under harsher conditions. The optimal design of an ADT aims to find an optimal test plan to evaluate the reliability or lifetime precisely by trading off between the utility and cost [30]. Bayesian ADT design treats model parameters as random variables to capture the parameter uncertainties by assigning prior distributions. Hence, compared to the traditional ADT optimal design, in which the crisp values are taken for the model parameters, Bayesian optimal design is a global optimal method.

Bayesian optimal design has been employed to the ADT design in the literatures, where the optimal design result depends on the specified utility objectives, such as minimizing the expected pre-posterior variance of the quantile life [19], maximizing the estimation precision of a specified failure-time distribution quantile [27], minimizing the dispersion of the estimated accelerated factors by M-optimality criterion [33], maximizing the Kullback-Leibler (KL) divergence (i.e., distance between prior distributions and posterior distributions) [16], and among others. In the aforementioned literatures, different optimization objectives have been selected in the Bayesian ADT design, and these objectives describe the utility of an ADT from the different aspects of evaluation precision or information gain. KL divergence expresses the information gain provided by an ADT, quadratic loss function [24] measures the evaluation precision of the quantity of interest, and Bayesian alphabet optimal [26] captures the evaluation precision of model parameters. In addition, the practical ADT design is often constrained by the total budgeted cost. However, such optimal objectives and the given budget concern cause the confusion to the engineers: how to choose the right utility for an ADT? In this study, multi-objective optimization methodology is utilized to solve this problem by generating a Pareto-optimal frontier of solutions with the consideration of the dominant optimality among several optimization objectives.

To the best of our knowledge, there is limited publication focusing on the multi-objective ADT design. Marseguerra et al. [22] proposed a formulation of two-objective ADT optimal design problem which optimized both the estimation accuracy of the failure time distribution

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percentiles and the testing cost. The two-objective programming was solved by a multi-objective genetic algorithm. However, there is only one utility function (i.e., quadratic loss function) taken into account, which implies that only the evaluation accuracy of quantile life was emphasized. The multi-objective optimal method has also been utilized in accelerated life test (ALT) design, which is different from ADT. The time-to-failure data are collected in ALT, while degradation data in ADT. Srivastava and Mittal [29] proposed a multi-objective optimal formulation for a ramp-stress ALT, and the optimal objectives were chosen to minimize the asymptotic variance of the maximum likelihood estimate (MLE) of the log values of quantiles 1%, 50% and 100%, respectively. Wu and Huang [34] investigated the ALT with two or more level constant-stress under competing risks and derived the optimal stress levels under D/A/variance-optimality criteria and the optimal sample allocations at each stress level under D/A/variance-optimality. By minimizing the weighted sum of the asymptotic variances of quantiles' MLEs under the constant stress level, the ramp rate of the accelerated stress were designed.

Constant stress ADT (or CSADT) and SSADT are two types of ADT that have been extensively developed. In particular, SSADT has been recognized as one of the most well-performed stress loading methods to shorten testing duration, with which many studies have been reported [11,12,20,21,31,32,39], and among others. Hence, the stress loading method we focus on is step stress. With the considerations of the essence of the quadratic loss function, KL divergence, and the given budget of SSADT, a multi-objective formulation of the SSADT plan design problem will be proposed in this study, and it aims to provide the optimal testing plans with regard to both the evaluation precision and the testing information gain.

Over the past two decades, numerous multi-objective evolutionary algorithms [5,9,28] have been developed. An overview and tutorial could be found in [14]. However, main criticisms of these MOEAs were raised over the recent years: (i) high computational complexity of non-dominated sorting; (ii) lack of elitism; (iii) need for specifying the sharing parameter share. All the above mentioned three drawbacks were well addressed by NSGA-II [3], which has been experimentally proved to outperform many MOEAs. Numerous studies [4,8,13,18,25] have proved that NSGA-II is an effective approach which captures a global search space and obtains well-distributed Pareto frontier. Therefore, in this study, NSGA-II will be used to generate a Pareto-optimal set of solutions to ADT.

The motivation of this study is to propose a Bayesian SSADT multi-objective design method with three objectives to be simultaneously, i.e., maximizing KL divergence, minimizing quadratic loss function, and minimizing testing cost. Section 2 gives some necessary preliminaries of IG process, SSADT settings, and Bayesian inference for model formulation. Section 3 presents the methodology framework of the proposed Bayesian optimal design for SSADT with IG process. Section 4 employs NSGA-II to iteratively find a Pareto-optimal set of solutions to the formulation of Bayesian SSADT multi-objective design problem. In Section 5, data envelopment analysis (DEA) is proposed to pruning the Pareto solutions. In Section 6, the effectiveness of the proposed Bayesian SSADT multi-objective design method and the solving procedure are illustrated with case studies on electrical connectors. Finally, Section 7 concludes the study and proposes potential directions for future research.

2. Preliminaries

In this section, necessary preliminaries of the IG process, SSADT settings, and Bayesian inference are pre-given for SSADT multi-objective model formulation in what follows.

2.1. IG process in SSADT

In this study, we assume that the degradation path of a product satisfies an IG process if it has the following three properties: (i) $Y(0)=0$ with probability one; (ii) $Y(t)$ has independent increments, i.e., $Y(t_2)-Y(t_1)$ and $Y(t_4)-Y(t_3)$ are independent, for $0\le t_1<t_2\le t_3<t_4$; (iii) Each degradation increment follows an IG distribution $\Delta Y(t)\sim \mathcal{IG}(\mu\Delta\Lambda(t),\lambda\Delta\Lambda(t)^2)$, $\mu>0$, $\lambda>0$, $\Lambda(t)$ is a given monotone increasing function of time t with $\Lambda(0)=0$, and $\Delta\Lambda(t)=\Lambda(t+\Delta t)-\Lambda(t)$.

Definition 1. (Chhikara, [2]) For any $x>0$, the probability density function (PDF) of $\mathcal{IG}(p,q)$ with mean p and variance p^3/q (p, $q>0$) is defined by

$$f_{\mathcal{IG}}(x;p,q)=\sqrt{\frac{q}{2\pi x^3}}\cdot\exp\left[-\frac{q(x-p)^2}{2p^2x}\right]. \tag{1}$$

Then the degradation process of a product can be characterized by $Y(t)\sim\mathcal{IG}(\mu\Lambda(t),\lambda\Lambda(t)^2)$ with the mean and variance as $\mu\Lambda(t)$ and $\mu^3\Lambda(t)/\lambda$, respectively. Substituting $p=\mu\Lambda(t)$ and $q=\lambda\Lambda^2(t)$ into Eq. (1) yields the expression of the PDF of $\mathcal{IG}(\mu\Lambda(t),\lambda\Lambda^2(t))$ rewritten by

$$f_{\mathcal{IG}}(x;\mu,\lambda)=\sqrt{\frac{\lambda(\Lambda(t))^2}{2\pi x^3}}\cdot\exp\left[-\frac{\lambda(x-\mu\Lambda(t))^2}{2\mu^2x}\right] \tag{2}$$

where μ is a parameter related to the degradation rate of a product, which is a function of the accelerating variable S; that is, $\mu(S)$ (an acceleration model), which can be written by

$$\mu(S)=\exp\left[a+b\varphi(S)\right] \tag{3}$$

where a and b the estimated parameters based on SSADT.

In this study, linear normalization method is applied to standardize the stress level. Let S_0 and S_H be the normal stress level and the highest stress level in ADT, respectively. Then the standardized function $\varphi(S)$ can be written as $\varphi(S)=\left(\xi(S)-\xi(S_0)\right)/\left(\xi(S_H)-\xi(S_0)\right)$, where $\xi(S)$ represents a pre-given function of different accelerating variable S. For example, $\xi(S)=1/S$ if the accelerating variable is temperature, and $\xi(S)=\ln(S)$ if the accelerating variable is electricity.

In addition, λ in Eq. (3) is a constant. If there are K accelerating variable levels in an ADT, then $\lambda_1=\lambda_2=\cdots=\lambda_K$. It is appropriate to assume $\Lambda(t)=t^\beta$ with $\beta>0$ [24], because when $0<\beta<1$, the trend is convex; when $\beta=1$, the trend is linear; and when $\beta>1$, then trend is concave.

The product fails when $Y(t)$ reaches a pre-given threshold level Y_D, and the associated first-passage-time is denoted by T_D. As the path of the IG process is strictly increasing, the cumulative distribution function (CDF) of T_D can be expressed as

$$F_{Y_D}(t)=P(Y(t)\ge Y_D)=\Phi\left[\sqrt{\frac{\lambda}{Y_D}}\left(t^\beta-\frac{Y_D}{\mu}\right)\right]-\exp\left(\frac{2\lambda t^\beta}{\mu}\right)\cdot\Phi\left[-\sqrt{\frac{\lambda}{Y_D}}\left(t^\beta+\frac{Y_D}{\mu}\right)\right]. \tag{4}$$

When both $\mu\Lambda(t)$ and t are large, $Y(t)$ is approximately normally distributed with mean $\mu\Lambda(t)$ and variance $\mu^3\Lambda(t)/\lambda$ (See e.g., [38]). Therefore, the CDF of Y_D can be approximately written by

$$F_{Y_D}(t)=1-\Phi\left[\frac{Y_D-\exp(a+b\varphi(S)t^\beta)}{\sqrt{\exp(a+b\varphi(S))^3t^\beta/\lambda}}\right]. \tag{5}$$

Based on Eq. (5), the p-quantile lifetime of Y_D can be easily obtained by

$$t(p)=\left[\frac{\mu}{4\lambda}\left(z_p+\sqrt{(z_p)^2+4Y_D\lambda/\mu^2}\right)^2\right]^{\frac{1}{\beta}} \tag{6}$$

where z_p is standard normal p-quantile.

In addition, we assume that the four parameters (i.e., a, b, λ and β) in Eq. (5) are mutually independent and constitute the parameter vector $\theta=(a,b,\lambda,\beta)$. Since the historical information and experts' knowledge are available before the implementation of SSADT, θ can be treated as a vector of random variables in Bayesian method to quantify these information and knowledge's contribution. It is known that the degradation increment x follows an IG distribution, and λ and β should be positive; therefore, common positive distributions (i.e., Gamma, Lognormal and