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# Performability analysis of multi-state series-parallel systems with heterogeneous components

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ABSTRACT

Motivated by many networked computing systems used in data centers, Grid or Cloud computing infrastructures, and spatially distributed wireless sensor networks configured in aerospace and military industries, this paper considers a multi-state series-parallel system where $n$ heterogeneous components can possess multiple states with different performance rates. Performability is concerned with the probability of such a system performing at a cumulative system performance $W$ characterized in terms of the minimum of all cumulative subsystem performance $G_j$ $(1 \leq j \leq m)$, $G_j$ is the sum of the performance rates of components belonging to the $j$th subsystem. This paper proposes a multi-valued decision diagram (MDD)-based approach to model and evaluate the concerned performability. A single compact MDD model is constructed by sharing all isomorphic model structures involved in different cumulative system performance. The MDD model can be reused when different versions of component state probability distributions are evaluated, thus it can facilitate solutions to further problems that require executing numerous iterations of performability evaluations, for example, the redundancy and/or reliability optimization problems.



## 1. Introduction

The typical architecture of a series-parallel system consists of $m$ subsystems connected in series, and in each subsystem there are $k$ components connected in parallel. The performance rate of a subsystem can be calculated based on the individual performance rates of its $k$ parallel components, while the performance rate of the entire system can be calculated based on the performance rates of its $m$ series subsystems. The calculation of subsystem/system performance rate strictly depends on the physical nature of the performance measure and on the nature of the interaction of system components. Following [1], when the performance measure is productivity or capacity, the subsystem performance rate is the sum of the performance rates of its parallel components while the system performance rate is the minimum of all subsystem performance rates, i.e., the subsystem with the least performance rate becomes the bottleneck of the system. For example, in a data center, the set of computing nodes plugged into the same switch can be treated as a subsystem and a computing task performed across multiple subsystems requires that each subsystem shall maintain a cumulative performance no less than a given demand $W$. More examples of series-parallel systems (continuous materials or energy transmission systems, manufacturing systems, power generation systems) can be found in [2,3].

When binary-state components are considered, every component in the series-parallel systems is defined by its performance rate and reliability function. The components exist in only two states viz. (i) functioning with nominal performance rate, and (ii) down with complete failure. System reliability evaluation and optimization have been formulated and solved for these systems [4,5].

In many practical series-parallel systems, the components can take multiple different states: from complete failure up to perfect functioning. For example, the computing nodes configured into modern networked computing systems typically exhibit more than two states correspond to different performance rates or computing powers. Consider a computing node with a multi-core processor, the failure of one or multiple cores causes degraded performance or computing power of this node [6]. Similarly, the failure of partial modules in a high-end computing node with multiple memory or storage modules can cause performance degradation [7].

The first attempt to introduce the concept of multi-state components was made in the 1970s [8]. When applied to systems consisting of multi-state components, reliability is a measure of the ability of a system to

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**Nomenclature**

| | |
|---|---|
| $n$ | number of components in the system |
| $m$ | number of subsystems connected in series |
| $k$ | number of parallel components in a subsystem |
| $d_i$ | number of states of component $C_i$ |
| $p_{i,r}$ | probability that component $C_i$ is in state $r$ |
| $g_{i,r}$ | performance rate of component $C_i$ in state $r$ |
| $X_i$ | the state variable for component $C_i$ |
| $\boldsymbol{H_j}$ | random vector representing states of $k$ components in the $j$-th subsystem |
| $\boldsymbol{h_j}$ | realization of random vector $\boldsymbol{H_j}$ |
| $\boldsymbol{Y_l}$ | random vector representing states of $l$ components numbered from 1 to $l$, $1 \le l \le n$ |
| $\boldsymbol{y_l}$ | realization of random vector $\boldsymbol{Y_l}$ |
| $\boldsymbol{Z}$ | random vector representing states of $n$ components in the system |
| $\boldsymbol{z}$ | realization of random vector $\boldsymbol{Z}$ |
| $G_j$ | cumulative subsystem performance of the $j$th subsystem |
| $W$ | cumulative system performance |
| $\Phi_w$ | probability that the system performs with the cumulative system performance $W$ |

meet a demand. For example, in a data center, it is the ability of the computing system to provide an adequate supply of computing power. Ushakov [9] introduced the concept of Universal Generating Function (UGF), which was later used to evaluate multi-state series-parallel system reliability [10]. Levitin [11] extended the UGF method to incorporate common-cause failures into multi-state series-parallel system reliability estimation, and the proposed technique allows the reliability function of multi-state series-parallel system to be obtained using a straightforward numerical procedure. Recently, to model many complex industrial systems having several operation phases, the dynamic behavior in the multi-state series-parallel system has been investigated [12]. Specifically, Markov process is used to model the dynamics of system phase changing and component state changing, and UGF is used to build system reliability function from the system phase model and the component models. In [13], the component allocation and maintenance has been considered simultaneously in order to minimize the total maintenance cost subject to the pre-specified system reliability requirement. An algorithm based on UGF is suggested to evaluate the system reliability. Moreover, significant research effort has been devoted to the development of meta-heuristic methods for handling the redundancy optimization problem in multi-state series-parallel systems. Ouzineb et al. [14] and Taboda et al. [15] combined UGF with Tabu Search (TS) and Genetic Algorithm (GA) techniques respectively to handle redundancy optimization in homogeneous multi-state series-parallel system. Tian et al. [16] presented an optimization model for a multi-state series-parallel system to jointly determine the optimal component state distribution and optimal redundancy for each stage. The physical programming approach was used to model and solve this reliability-redundancy optimization problem with two design objectives: system utility and system cost. The formulated single-objective nonlinear optimization model is then solved using GA. Liu et al. [17] conducted the redundancy optimization and maintenance planning for multi-state series-parallel systems in a joint manner.

Different from reliability analysis, performability analysis is concerned with the system performance distribution [18]. For the data center example, various computing tasks might have different performance demands and it is required to calculate the probabilities for each possible performance demand. Basically, performability analysis involves generating and solving a number of reliability problems separately. Very often, models for those reliability problems share common substructures, but they still need to be stored and evaluated multiple times. To improve the computational efficiency and reduce the storage requirement, these identical model structures should not be created and evaluated repeatedly.

When the constituent components are homogeneous, sharing identical model structures can be achieved relatively simple. Researchers have provided fast algorithms for calculating the state distribution of this sort of multi-state systems having series-parallel structures, for example, the finite Markov chain imbedding approach proposed in [19] and the recursive algorithm proposed in [20]. In practice, the assumption that all components of a subsystem must be homogeneous leads to an increase in the design costs of the system and prevents the attainment of higher levels of reliability; obviously, it is possible to design a system that provides the desired reliability with a lower cost, using non-homogeneous components instead of homogeneous ones. In real world, components in a system can be heterogeneous because they might be provided by different suppliers or have different product/model types or be installed under different natural or geographic environments. Compared with homogeneous systems, performability analysis of heterogeneous system is much more complex.

Recently, a suite of new approaches based on the state-of-the-art data structure called multi-valued decision diagram (MDD) has been proposed to address the challenge in performability analysis of multi-state heterogeneous systems [21]. MDD can represent a multi-valued logical function as a rooted, directed acyclic graph in the form that is both canonical and compact through two reduction rules "merging isomorphic subtrees" and "deletion of useless nodes" [22]. Empirical study results show that MDD approaches have much lower computational complexity in both model generation and evaluation than that of traditional methods [23–25].

In this paper, we will make a new contribution by modeling a multi-state series-parallel system with a single MDD model for all possible cumulative system performance. This MDD model is compact by sharing all isomorphic model structures involved in different cumulative system performance. The unique MDD model is then evaluated to obtain system performability measures and the performability evaluation procedure based on MDD has the complexity as a linear function of the number of MDD nodes. Due to the reuse of MDD model, the performability analysis time will be reduced as the model evaluation time when different versions of component state probability distributions are considered. Thus, the proposed MDD method can be very critical for applications that require many times of evaluations, for example in the optimization problems of redundancy or reliability allocation as compared to the existing UGF method.

The remainder of this paper is organized as follows: Section 2 defines the specific problem to be addressed in this work. Section 3 is devoted to the development of the MDD-based approach to model and evaluate performability of a multi-state series-parallel system with heterogeneous components, and the complexity analysis of the constructed MDD models is analyzed. Numerical experiments are presented to illustrate the applicability of the proposed method in Section 4. A conclusion and our future works are presented in Section 5.

## 2. Problem statement

In this work, the cases of multi-state series-parallel systems with heterogeneous components are investigated. Specifically, a multi-state series-parallel system consists of $n$ multi-state components. Each component $C_i$, $1 \le i \le n$, can be in one of $d_i$ different states. Each state $r \in \{0, 1, \ldots, d_i - 1\}$ of component $C_i$ is characterized by its probability $p_{i,r}$ and performance rate $g_{i,r}$.

$$\sum_{r=0}^{d_i-1} p_{i,r} = 1 \tag{1}$$