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On the use of AR models for SHM: A global sensitivity and uncertainty analysis framework

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A B S T R A C T

This paper proposes a complete sensitivity analysis of the use of Autoregressive models (AR) and Mahalanobis Squared Distance in the field of Structural Health Monitoring (SHM). Autoregressive models come from econometrics and their use for modelling the response of a physical system has been well established in the last twenty years. However, their aware application in engineering should be supported by knowledge about how they describe phenomena which are well defined by physics. Since autoregressive models are estimated by a least square minimization, statistical tools like Global Sensitivity Analysis and uncertainty propagation are powerful methods to investigate the performance of AR models applied to SHM.

These methodologies allow one to understand the role of the uncertainty and uncorrelated noise by a rigorous approach based on statistical motivations. Moreover, it is possible to quantify the link between the mechanical properties of a system and the AR parameters, as well as the Mahalanobis Squared Distance. By fixing a factor prioritization among the variables of a AR model, it is possible to understand which are the parameters playing a main role in damage detection and which type of structural changes is possible to efficiently detect.

1. Introduction

The importance of improving the understanding of the performance of structures over their lifetime with information obtained from Structural Health Monitoring (SHM) has been widely documented. The assessment of structural reliability is strictly connected to the quality of information provided by a damage detection process [1–6]. The diagnosis of in-service structures on a continuous real-time basis is of primary importance for aerospace, civil and mechanical engineering. The advantages of Structural Health Monitoring (SHM) are optimal use of a structure, reduced downtime and avoidance of catastrophic failures, moreover it can drastically change the planning of maintenance service with several economic benefits [7]. There are many potentially useful techniques to achieve these aims, and their applicability to a specific situation depends on the size of the acceptable critical damage for a structure.

As mentioned in the given reference, the problem of damage detection has a hierarchical structure. At the lowest level, it is required to recognize damage has occurred or not. At the highest level, damage location and size must be identified for a proper estimation of the residual structure life. One of the most promising approaches to damage identification is based on pattern recognition [8]. Data are measured from a structure and converted, by a process of feature extraction, into a representation where variations due to damage are highlighted.

Vibration based methods have been widely used for identification of various types of damages for several real and laboratory structures. The methods relying on vibration response -also known as Output-Only methods –represent an important category within the vibration based methods for Structural Health Monitoring (SHM). Their use is highly important for in-service structures such as bridges, aircrafts, naval vehicles and others, where the excitation signal is not available.

In the last twenty years, the scientific community focused its attention on exploitation of different types of autoregressive models and features for SHM. In [9] *Sohn* et al. studied *AR* models and features based on analysis of residuals (X-Chart, S-Chart, EWMA). In [10] *Carden* et al. applied the more complex *ARMA* models to SHM application; the approach has been validated with experimental data taken from Z24 bridge. *Sohn* et al. in [11] proposed a linearized version of *ARMA,* the *AR-ARX* model. The applicability of this approach has been demonstrated with an experimental setup based on an eight degree of freedom mass-spring system. Features based on residuals often assume a Gaussian distribution of sample data sets. This assumption might be misleading, making SHM algorithms less efficient. *Worden* et al. in [12] tried to overcome this problem proposing a more sophisticated data processing called *sequential probability ratio test* (SPRT), which relies on the analysis of extreme value statistics. In [13] *Yao* et al. proposed a comparison between sev-

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Nomenclature

X_t, X_{t-1}, X_0	Signal sample at time $t, t-1, 0$
ϕ_1	Autoregressive parameter of AR(1) model
a_t	Residual between model prediction and signal output at time t
q	Order of the AR(q) model
D_m	Mahalanobis square distance
$[\phi]$	qxp Matrix of Autoregressive reference dataset. p is the number of set acquired
ϕ_i	Generic i-th AR parameter of the model
$\{\phi_\mu\}$	$qx1$ Vector of the means of the Autoregressive reference datase
$[S]$	qxq Covariance Matrix of the Autoregressive reference dataset
$\{\hat{\phi}\}$	$qx1$ Vector of the Autoregressive parameters assumed for the outlier analysis
Y	General output quantity
X_i	General i-th input quantity
$V(Y)$	Total variance of the Y output
$V(E(Y\|X_i))$	Conditional variance of the Y output with respect X_i input
$V(E(Y\|X_{\sim i}))$	Conditional variance of the Y output with respect all inputs except X_i
S_{X_i}	First order global sensitivity index
S_{X_i,X_j}	Second order global sensitivity index
S_{T_i}	Total sensitivity index
$[M]$, $[D]$, $[K]$	Mass matrix, damping matrix, stiffness matrix of a generic mechanical system
m, h, k	Mass, damping ratio and stiffness of 1 d.o.f. mechanicals system
σ_u	Standard deviation of Gaussian white noise excitation
$U(\sim)$	Uniform distribution

eral pattern recognition algorithms using autoregressive models. Moreover, they introduced a new feature extraction technique, called *Cosh spectral distance* (COSH) and validated it with several experimental data.

Among all the possible strategies for time series modelling, the use of pure AR models is very common. Basically, because the identification of the model is made by a simple least squared minimisation, which requires few computing efforts to be performed, and the uncertainty of the model is usually low. However, pure AR models are only-pole functions and can represent the response of complex systems just by an approximation. The consequences are spurious poles that must be introduced in the model in order to follow the response of the mechanical system, although it depends also on the zeros of the frequency response function of the physical model [14]. The popular application of AR models to SHM relies on their reliable identification of the mechanical properties of a system; however, the main property that should be taken into account, in a SHM context, is their sensitivity to a change of the system they are representing. Since AR models are made by physical and spurious poles, their sensitivity is not a trivial issue. The AR parameters of a model are usually used all together to assess the healthy status of a system, for instance computing a Mahalanobis Square Distance. However, as it will be shown, only few of them are strongly linked to the physical properties of the system; so that, their behaviour is not strictly depending on the physical response they are modelling and to its changes.

Scientific literature lacks examples for the explicit propagation of measurement uncertainty and Global Sensitivity Analysis for damage detection algorithms. *Yao* et al. in [14] proposed a formulation for the sensitivity of Mahalanobis Squared Distance and COSH distance with respect to both stiffness reduction and measurement noise level. However, the analysis is based only on an analytical study of the issue in place of a statistical framework. In that work, simulation results and theoretical analysis show some differences due to the approximation adopted for the extrapolation of the sensitivity expression. In [15] *Roy* et al. provided a mathematical formulation to establish the relation between the change in an ARX model coefficients and the normalized stiffness of a structure. The reason behind the choice of ARX model in place of a standard AR model is the coefficients of the ARX model can have a direct correlation with structural stiffness. Such a correlation is however not established for standard AR model.

This work focuses on an accurate analysis of the uncertainty related to vibration-based method. Specifically, it focuses on the use of pure autoregressive models and Mahalanobis Squared Distance, among the most widely adopted approaches in vibration-based methods. Generally, this approach could be extended to any kind of damage feature to quantify its sensitivity to the changes of a system.

The contribution of this paper is the attempt of covering the lack of uncertainty assessment in the SHM literature, performing an Uncertainty Propagation Analysis (UP) and a Global Sensitivity Analysis (GSA) of AR models and Mahalanobis Squared Distance. As it will be proved, a rigorous analysis will demonstrate that pure AR models may hide some weaknesses that could have strong consequences on their feasibility to SHM. This paper will give some guidelines about the variables that strongly affect the performance of AR models for damage detection and about the type of structural changes that might be detected with confidence. The conclusion will be fundamental to those who want to use AR models as tools to get information to predict the safety of aging structures over their service life.

The paper is structured as follows. In Section 2 the background theory of Autoregressive models and Mahalanobis Squared Distance are briefly exposed, in the context of damage detection. The Analysis of Variance is introduced in Section 3. The design of the simulation is discussed in Section 4. Finally, the results of the Analysis of Variance of the AR model and Mahalanobis Squared Distance applied to damage detection are presented and commented in Section 5.

2. Autoregressive models

In the next paragraphs, the background theory of AR models and Mahalanobis Squared Distance is briefly introduced, putting them in the context of dynamic system identification. The reader is asked to refer to a complete background theory provided by the following reference [16].

2.1. Description of a system response function by autoregressive models

Autoregressive models were developed in econometrics as a representation of time-varying processes, in which the output variable depends linearly on its own previous values and on a stochastic term. Nowadays they are used in a wide variety of different fields, for instance Structural Health Monitoring (SHM). Autoregressive models can be implemented to represent the dynamic response of structures. Through these models, it is possible to describe a time series with a lower number of data, which are the parameters of the AR model.

To introduce autoregressive models, let us consider a linear mechanical system. Let us call $u(t)$ the input of a system and $x(t)$ the output. Usually the input could be either a deterministic variable or a stochastic one. In a civil or mechanical structure, which works under operational conditions, the excitation can be described as a random force. Under some strong assumptions [17], operational modal analysis considers the random force as a Gaussian process. In a real case, these assumptions are quite well respected if the data are averaged over a long enough time window. Therefore, under this condition, $u(t) \sim NID(0, \sigma_u^2)$.

Now it is possible to recall the link between a generic dynamic system and an AR model. For sake of simplicity, let us start with the autoregressive model of the first order $AR(1)$, which represents a dynamic response of the first order in the discrete time domain:

$$\boldsymbol{X_t = \phi_1 X_{t-1} + a_t} \tag{1}$$