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A universal generating function-based multi-state system performance model subject to correlated failures

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ABSTRACT

Multi-state system (MSS) reliability modeling is a paradigm that allows both systems and components to exhibit more than two performance levels. While several researchers have introduced correlation or dependence into MSS models to assess its negative influence on performance and associated measures, these methods exhibit complexity that is exponential or worse in the worst case. To overcome this limitation, this paper proposes an extension to the discrete universal generating function approach for MSS to allow correlation between the elements comprising a multi-state component. We subsequently generalize to the continuous case and allow failures to follow any life distribution. The approach possesses an analytical form and therefore enables efficient performance and reliability assessment as well as sensitivity analysis on the impact of correlation. This sensitivity analysis can be applied to a wide range of measures including performance, reliability, the density function, hazard rate, mean time to failure, availability, and mean residual life. The approach is illustrated through a series of examples, demonstrating the efficiency of the approach to assess performance and reliability as well as to conduct sensitivity analysis. The results indicate that the approach can identify the impact of correlation on performance, reliability, and the many measures of interest.

1. Introduction

Multi-state system performance and reliability analysis [1] has received substantial attention very recently [2–5] as a valuable generalization of traditional binary state reliability models of component-based systems. The MSS paradigm can model systems composed of components that exhibit three or more levels of performance and are therefore suitable for modeling modern systems such as scientific workflows in a cloud computing environment [6,7], where the size and the availability of server farms determine the performance of a computation. Similar to binary state systems, MSS are also susceptible to various forms of dependent and correlated failures, where failures of the elements comprising an MSS component[1] experience simultaneous or cascading failure. These dependent and correlated failures can have several negative impacts, including lower performance, reliability, and earlier mean time to failure (MTTF), necessitating that reliability engineering managers take steps to mitigate the possibility of these negative consequences. The vast majority of MSS research that consider dependence utilizes methods such as common cause failure (CCF) [8] and Markov modeling [9]. However, the CCF approach introduces an exponential number of parameters in the worst case, while the Markov method suffers from the state space explosion problem. Thus, simpler analytical methods to efficiently model and assess the impact of correlated failures on the performance and reliability of MSS could further enhance the applicability of the MSS paradigm to modern systems.

One of the earliest works on multi-state systems is that of Barlow and Wu [10] who defined a system state function for coherent systems with multi-state components and investigated its properties. A recent survey of multi-state system reliability modeling and evaluation by Yingkuia et al. [11] notes that virtually all MSS methods are relatively straightforward extensions of binary state methods, including the assignment of Boolean variables to each MSS component state [12] enabling reduction to a binary system, stochastic processes [13–15], the universal generating function(UGF) [16], Monte-Carlo simulation [17–19], and recursive algorithms [20,21]. Yingkuia et al. [11] also note that many or all of

Abbreviations: CCF, common cause failure; MTTF, mean time to failure; MTTR, mean time to repair; MRI, mean residual life; MSS, multi-state system; p, parallel system; s, series system; UGF, universal generating function

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[1] This paper does not use the terms element and component interchangeably. Elements are internal to MSS components. Reliable elements determine the performance of an MSS component. Since components considered in this paper are multi-state, MSS is often omitted before the word component for brevity.

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Nomenclature

Notation

X	set of systems $x \in \{s, p\}$
n	number of components
k_j	number of distinct non-zero performance levels of component j
g_{jh}	performance of component j in state h
g_j	vector of performance levels of component j
G_j	random variable of performance level of component j
p_{jh}	probability component j exhibits state h
p_j	vector of component j states
L^n	cross product of component performance levels
$K+1$	number of distinct system performance levels
M	set of system performance levels
ϕ	mapping of component performances to system performance, $L^n \rightarrow M$
W	random variable of system performance level
w_i	system performance level in state $i \in \{0, 1, \ldots, K\}$
q_i	probability system exhibits performance level w_i
$R^x_{w_{\min}}$	probability system x performance meets or exceeds minimum threshold $w_{\min}$
u_i	u-function components i
$\otimes_{\phi_x}$	composition operator of u-functions for system $x \in X$
ρ	component correlation
$R^x_i(t)$	reliability of system x for threshold $w_{\min} \geq i$ at time t
$f^x_i(t)$	density function of system x for threshold $w_{\min} \geq i$ at time t
$h^x_i(t)$	hazard rate of system x for threshold $w_{\min} \geq i$ at time t
$MTTF^x_i$	MTTF of system x for threshold $w_{\min} \geq i$
A^x_i	availability of system x for threshold $w_{\min} \geq i$
$MRL^x_i(t)$	MRL of system x for threshold $w_{\min} \geq i$ at time t

the papers they surveyed assume component failures are independent.

Multi-state system reliability models considering correlation include the work of Veeraraghavan et al. [22] who proposed a combinatorial algorithm to assess the performance and reliability of a coherent repairable system composed of multi-state components, which allows interdependent component state transitions within a system where each component can exhibit a different number of performance levels. Zang et al. [23] developed a method based on binary decision diagrams where boolean variables represent the state of components and a series of multi-state fault trees represent the multi-state system. Trivedi et al. [24] presented multi-state availability models using three analytic techniques: (1) continuous time Markov chains, (2) stochastic reward nets, and (3) multi-state fault trees and performed a comparative performance analysis of a system where component failures can be statistically dependent.

A large body of work on MSS utilizes the common cause failure [8], including Levitin [25] who extended the UGF approach to include common cause failures [8], while in Ref. [26], he used common cause failures to characterize statistical dependence among failures of components with multiple levels of protection. Korczaka et al. [27] modeled the survivability of series–parallel MSS with multiple levels of protection through a composition of Boolean and UGF techniques. Levitin et al. [28] presented an algorithm to evaluate the performance distribution of series–parallel MSS where CCF are caused by the propagation of failures among system elements. Levitin et al. [29,30] also modified the reliability block diagram (RBD) method to evaluate the reliability and performance measures of multi-state series–parallel systems with performance-dependent fault coverage. They also introduced a new model of fault level coverage for MSS, where the effectiveness of recovery mechanisms depends on the coexistence of multiple faults in related elements. In Ref. [31], Levitin et al. considered the performance evaluation of series–parallel MSS with propagated failures and imperfect protections. Xing and Levitin [32] proposed a separable and combinatorial methodology for the reliability analysis of MSS subject to propagated failure with global effect and failure isolation according to functional dependence.

Additional contributions by Levitin [33] extended the UGF approach for MSS to the case where element failures exhibit unilateral dependency, while in Ref. [34], Levitin presented a UGF model of multi-state systems in which some groups of elements can be affected by uncovered failures that cause outages of the entire group. Peng et al. [35] considered a UGF-based multi-state series–parallel system with two types of parallelization, namely redundancy and work sharing. The authors extend the problem of finding the optimal balance between redundancy and task sharing in multi-state systems with uncovered failures to the cases of multi-fault coverage. More recently, Dao et al. [36] studied the reliability analysis of MSS with s-dependent components, combining techniques from stochastic processes and a modified UGF to evaluate the system reliability and verify the combined approach with Monte Carlo simulation.

This paper is a direct extension of Levitin's discrete UGF approach for multi-state system reliability [33]. We extend this fundamental approach to the case where the failures of elements comprising the components of a MSS are identically distributed but correlated and study the impact of correlated failures on the performance and reliability of series and parallel MSS. We subsequently generalize the discrete UGF approach with correlated identical components to the time varying case and assess the impact of correlation on the reliability, density function, hazard rate, mean time to failure, availability, and mean residual life (MRL). The illustrations demonstrate that the approach can identify the impact of correlation on series and parallel MSS for various minimum performance levels.

The proposed approach can be distinguished from previous research which utilizes techniques such as CCF and Markov dependence to characterize the correlation between the failures of the elements of an MSS component or correlation between the performance levels of MSS components. The CCF approach introduces an exponential number of parameters and the Markov method is subject to the state space explosion problem. The proposed approach can also be distinguished from the authors' previous research on systems composed of correlated identical components [37], which was limited to reliability and sensitivity analysis of binary state systems composed of binary state components and only considered a subset of the reliability measures considered here. The present paper demonstrates the broad applicability of correlated identical components in the context of the universal generating function approach to multi-state system performance and reliability assessment.

Correlated identical components possess an analytical form which promotes computationally efficient point calculations. Furthermore, the analytical nature of the expressions enables efficient sensitivity analysis on the explicit correlation parameter, thereby promoting intuitive assessment of the impact of correlation on a range of reliability measures. The approach can thus enable quality management, providing objective methods to assess the impact of