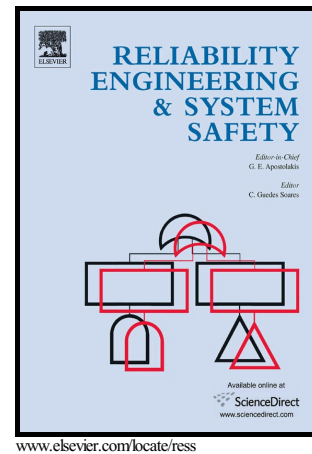# Author's Accepted Manuscript

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Five and Four-parameter Lifetime Distributions for Bathtub-shaped Failure Rate Using Perks Mortality Equation

Hongtao Zeng, Tian Lan, Qiming Chen

Abstract—**Two lifetime distributions derived from Perks' mortality rate function, one with 4 parameters and the other with 5 parameters, for the modeling of bathtub-shaped failure rates are proposed in this paper. The Perks' mortality/failure rate functions have historically been used for human life modeling in life insurance industry. Although this distribution is no longer used in insurance industry, considering many nice and some unique features of this function, it is necessary to revisit it and introduce it to the reliability community. The parameters of the distributions can control the scale, shape, and location of the PDF. The 4- parameter distribution can be used to model the bathtub failure rate. This model is applied to three previously published groups of lifetime data. This study shows they fit very well. The 5-parameter version can potentially model constant hazard rates of the later life of some devices in addition to the good features of 4-parameter version. Both the 4 and 5-parameter versions have closed form PDF and CDF. The truncated distributions of both versions stay within the original distribution family with simple parameter transformation. This nice feature is normally considered to be only possessed by the simple exponential distribution.**

Index Terms—**Bathtub shape, hazard rate function, Perks, parameter estimation**

ACRONYMS

CDF	cumulative distribution function
HRF	hazard-rate function
MLE	maximum likelihood estimation
PDF	probability density function
Perks4	probability distribution derived from 4-parameter Perks hazard rate function
Perks5	probability distribution derived from 5-parameter Perks hazard rate function

NOTATION

$\gamma, \kappa, \mu, \beta, \alpha$ Perks5 parameters

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