Reliability Engineering and System Safety 144 (2015) 12–22

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Optimal backup frequency in system with random repair time

Gregory Levitin [a,b,*], Liudong Xing [c], Yuanshun Dai [a]

[a] Collaborative Autonomic Computing Laboratory, School of Computer Science, University of Electronic Science and Technology of China, China
[b] The Israel Electric Corporation, P.O. Box 10, Haifa 31000, Israel
[c] University of Massachusetts, Dartmouth, MA 02747, USA

ARTICLE INFO

Article history:
Received 12 August 2014
Received in revised form
14 June 2015
Accepted 18 June 2015
Available online 26 June 2015

Keywords:
Backup
Repair
Mission reliability
Mission cost
Mission time

ABSTRACT

This paper considers single-component repairable systems performing backup procedures to avoid repeating the entire work from scratch and thus facilitate fast system recovery in the case of failures. The mission succeeds if a specified amount of work can be accomplished within the maximum allowed mission time or deadline. The repair time is randomly distributed within a specified interval. Both failure and repair times are represented by known distributions. We first suggest a numerical algorithm to evaluate mission reliability, conditional expected cost and completion time of a successful mission. The backup frequency optimization problem is then formulated and solved for finding the inter-backup interval that maximizes mission reliability or minimizes expected mission cost while satisfying a desired level of mission reliability. Impacts of parameters including the maximum allowed mission time, data backup and retrieval times, repair and failure time distributions, and repair efficiency on mission reliability, cost and time as well as on the optimal solution are investigated through examples.

1. Introduction

A system is repairable if after its failure, the system can be recovered to some fully satisfactory performance level via maintenance activities such as replacement and part adjustment [1,2]. According to the degree to which the working condition of a system can be restored through maintenance, three types of repair models can be differentiated [3,4]: under the *perfect repair* model, the system is restored to an "as good as new" condition; under the *minimal repair* model, the system is restored to an "as bad as old" condition, that is to the same state as it was immediately before its failure; under the *general* or *imperfect repair* model, the system is restored to any condition between the former two cases.

Numerous research efforts have been expended in modeling and optimizing single-component or multi-component repairable systems under the different repair models. Renewal processes (in particular, homogeneous and non-homogeneous Poisson processes) [5,6], Markov chains [7–10], geometric processes [11–14], and Bayesian methods [15–17] are among the commonly-applied techniques for modeling the failure behavior of a repairable system. Diverse optimization problems have been formulated and addressed for repairable systems subject to different maintenance policies or behaviors. For example, replacement policy optimization problems have been solved for repairable systems with a repairman who can take multiple vacations [11,18], for systems under free-repair warranty [19], and for systems subject to waiting and repair times [20]. The optimal inspection scheme problems (periodic and non-periodic) have been addressed for repairable systems subject to hidden failures (failures that are not evident to operators and can only be rectified during inspections) or repairable systems with interactions between self-announcing hard failures and non-self-announcing soft failures [21–27]. A joint optimization problem has been solved in [28] for repairable systems with the objective to find the optimal number and schedule of preventive maintenance actions as well as the corresponding maintenance degrees. The joint optimal maintenance and warranty policy problem has been solved in [29] for repairable systems considering all phases of the system life cycle, where the optimal burn-in period, optimal preventive maintenance intervals and optimal replacement times are determined.

In spite of the rich literature on the modeling and optimization of repairable systems, to the best of our knowledge, none of those existing works has considered backup behavior as well as the related optimal backup frequency problem. For systems especially those used in computing-related applications, the backup mechanism is necessary to facilitate an effective system recovery or reconfiguration in the case of failures occurring. Specifically backup procedures are performed periodically to save data associated with the completed fraction of the mission task so that the

* Corresponding author.
E-mail addresses: levitin@iec.co.il (G. Levitin), lxing@umassd.edu (L. Xing).

http://dx.doi.org/10.1016/j.ress.2015.06.014
0951-8320/

Nomenclature

B	backup time
b	data retrieval time
$1(\cdot)$	unity function 1(FALSE)=0; 1(TRUE)=1
g	time needed to retrieve data after the a-th backup: $g=b\cdot 1(a>0)$
N	maximal possible number of repairs during the mission
τ	maximum allowed mission time
R	mission reliability
E	expected mission completion time
C	expected cost of a successful mission
θ_j, Ψ_j	conditional expected mission repair, operation time given the mission succeeds after j repairs
$\langle T_j, X_j, A_j \rangle$	event that the j-th failure happens at time T_j given the system spends time X_j in the operation mode and completes A_j backups before the failure
$Q_j(t,x,a)$	function representing a joint distribution of random values T_j, X_j and A_j
$q_j(v,w,a)$	probability that the j-th failure happens in time interval v and the system spends w intervals in the operation mode and completes a backups before the failure
D	random repair time
$d_{\min}$, $d_{\max}$	minimal, maximal possible realizations of D
H	maximum number of backup actions during the mission
$f(t)$, $F(t)$	*pdf* and *cdf* of system time-to-failure distribution
$\psi(t)$, $\Psi(t)$	*pdf* and *cdf* of repair time
W	pure mission time (time needed to complete the mission without backups and failures)
π	fraction of the entire mission task that should be backed up periodically (referred to as backup frequency parameter)
πW	time during which the system produces information that should be backed up
σ	time between the ends of two consecutive backup actions in the case of no failures: $\sigma=\pi W+B$
m	number of discrete intervals considered in the numerical algorithm
Δ	duration of a discrete time interval: $\Delta=\tau/m$
h	number of time intervals needed to complete the mission if no failures occur
$\lfloor x \rfloor$	floor operation that returns the maximal integer not exceeding x
Y_j	event that the mission is completed after j repairs
r_j	$\Pr(Y_j)$
φ_j, θ_j	expected mission operation, repair time given that j failures happen during the mission
η, β	scale, shape parameters of Weibull time-to-failure distribution
c_r, c_o	per time unit repair, operation cost
z	repair efficiency coefficient ($z=0$ corresponds to as good as new, $z=1$ corresponds to as bad as old)

failed system, upon being repaired, can resume the mission task from the latest backed up point instead of re-performing the entire mission task from the very beginning. In recent work [30], effects of periodic backups were first considered for modeling and evaluating mission reliability, expected mission time and cost of 1-out-of-N: G, cold standby systems. However, the model of [30] assumes that the system components are not repairable during the mission.

In this paper we make novel contributions by modeling single-component repairable systems subject to periodic backup actions, random failure and repair times, as well as real-time constraint on mission task completion. Three mission performance indices are evaluated, including mission reliability, conditional expected mission cost and completion time given a successful mission. The proposed evaluation algorithm is applicable to different repair models (perfect, imperfect, minimal) as well as arbitrary types of time-to-failure and time-to-repair distributions. The optimal backup frequency problem is formulated and solved for the considered repairable system where the optimal number of backups is identified for maximizing mission reliability or minimizing expected mission cost. Effects of different parameters on mission performance indices as well as on the optimal backup frequency solution are also investigated through examples.

The remainder of the paper is organized as follows. Section 2 presents the system model and formulation of the optimization problem addressed in this work. Section 3 discusses the evaluation of mission reliability, conditional expected cost and completion time of successful mission for real-time repairable systems subject to periodic backups and random repair times. Section 4 presents the discrete numerical evaluation algorithm based on the derivation in Section 3. Section 5 presents illustrative examples. Effects of various factors on mission indices and on the optimal backup frequency solution are investigated. Section 6 presents conclusions as well as directions of future work.

2. System model and problem description

2.1. System description

The system should accomplish a desired mission task within a time not exceeding τ, i.e., the system is a real-time system. The time required to accomplish the entire mission without backups or failures is W. Each fraction π of the entire mission task should be periodically backed up. Thus, the system conducts a data backup procedure after successfully performing the mission task during time πW. The total number of backup actions performed during a successful mission (i.e., backup frequency) is fixed and can be determined as:

$$H=\begin{cases}\lfloor 1/\pi \rfloor \text{ if } \pi\lfloor 1/\pi \rfloor<1\\ \lfloor 1/\pi \rfloor-1 \text{ if } \pi\lfloor 1/\pi \rfloor=1\end{cases}. \tag{1}$$

The second case in (1) is because when $\pi\lfloor 1/\pi \rfloor=1$, the last backup is scheduled to be performed at the end of the mission which is not necessary. Notice that π is referred to as a backup frequency parameter to be optimized in the optimization problems considered in this paper. Each backup action takes constant time B. Thus, the minimal time needed to complete the mission (given that no failures happen) is $HB+W$.

The system time-to-failure distribution is known and determined by the cumulative distribution function (*cdf*) $F(t)$. When the system fails, the repair procedure starts immediately. The repair time depends on external factors such as availability and efficiency of the repair manpower and equipment, and is randomly distributed within the interval $[d_{\min}, d_{\max}]$. Having the minimum possible repair time, the maximum possible number of failures that can occur in a successful mission can be obtained as:

$$N=\lfloor (\tau-HB-W)/d_{\min} \rfloor. \tag{2}$$