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Approximate methods for optimal replacement, maintenance, and inspection policies

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ABSTRACT

It might be difficult sometimes to derive theoretical and numerical solutions for analytical maintenance modelings due to the computational complexity. This paper takes up several approximate models in maintenance theory, by using the cumulative hazard function $H(t)$ and the newly proposed asymptotic MTTF (Mean Time to Failure) skilfully. We firstly denote by t_x the time when the expected number of failures is x. Using $H(t_x)=x$, we estimate failure times, model age and periodic replacements, and sequential imperfect maintenance. Motivated by the asymptotic method of computation of MTTF, we secondly model the expected cost rate for a parallel system when replacement is made at system failure, and give approximate computations for the sequential inspection policy. Optimizations of each model are obtained approximately in an easier way. When failure times have a Weibull distribution, it is shown from numerical examples that the obtained approximate optimal solutions have good approximations of the exact ones.

1. Introduction

Manufacturing systems with performance degradation and maintenance strategy are commonly encountered in practice. There have been many maintenance models in reliability, most of which are formulated stochastically, and are optimized analytically or simulated numerically by using algorithms [2,15,11,9].

However, it sometimes might be difficult to derive theoretical and numerical solutions for analytical maintenance modelings due to the computational complexity. One example is the sequential inspection policy [2], whose algorithm needs to make computations repeatedly until the procedure meets the required condition by adjusting the first checking time. To avoid this trouble, a nearly optimal inspection policy that depends on the parameter p was suggested [7]. However, to suppose the unit fails with constant probability p is too stronger to be applicable, even though this policy has been used for Weibull and gamma distribution cases [8,17]. Other works, such as an approximate solution of a maintenance policy for a system with multi-state components [4], an approximate inspection interval for production processes with finite run length [3], and approximations to determine the optimal replacement times of a sequential age replacement policy for a finite time horizon [6], have been discussed.

It has been well-known in reliability theory that the cumulative hazard function $H(t)$ represents the expected number of failures in the time interval $[0,t]$ [11]. On the other hand, the most concerns in reliability theory are to estimate the MTTF (Mean Time to Failure) and maintenance times of systems; however, when the system becomes more complex or larger sized, estimations become more difficult. One approximate analytical approach has been proposed to estimate a threshold maintenance policy for an n identical unreliable components system [1]. An asymptotic MTTF and approximate age replacement for a random-sized parallel system have been proposed recently [13].

Followed by the conference discussion [12], we use the cumulative hazard function $H(t)$ and the approximate computation of MTTF skilfully, and propose approximate methods to estimate failure times, and to optimize replacement, maintenance, and inspection policies. We show good approximations for the exact results in numerical examples when their failure times have a Weibull distribution as follows:

1. When failures occur at a non-homogeneous Poisson process and the unit undergoes minimal repair at each failure, it is of interest to observe the mean times of X_n between failures [11, p. 97]. We introduce $H(t_n)=n$ in which t_n $(n=1,2,\dots)$ is the time when the expected number of failures is n, and show that

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failure times X_n could be computed from a simpler equation form.

2. We introduce the mean rate of failures $H(T_x)=x(0<x\le 1)$ into age and periodic replacement policies. Computation of approximate T_x for age replacement is more easier but also is close to the exact T^*. For the periodic replacement, both approximate policies at time T and at number N of failures are obtained as one simpler equation form.
3. For the imperfect preventive maintenance policy [11], we obtain the approximate expected cost rate and its optimal maintenance number N^* and sequential maintenance times λT_k^*. It also shows numerically that the above approximate policies are close to the exact results.
4. Recently, an asymptotic method of computation of the MTTF and optimization of age replacement [13] for a random-sized parallel system are discussed. Using the approximate MTTF, we compute optimal number of units needed for a parallel system in an easier form, when replacement should be made after system failure.
5. Although improvements have been made to compute sequential inspection policies [10] which have been summarized in [11], it is still difficult to choose an appropriate ε to begin the algorithm. Motivated by the above approximate MTTF, we finally give approximate computations for the sequential inspection times.

There has been lack of research on the approximate methods for maintenance modelings and optimizations.

The remainder of this paper is organized as follows: Section 2 gives approximate x_k for the mean times of X_n between failures. The T_x and t_x^* are obtained as approximations of the exact optimizations for age and periodic replacement policies in Sections 3 and 4. Sequential imperfect maintenance times λT_k^* are derived in Section 5. An approximate number of units for a parallel system and sequential inspections times $\tilde{T}_k$ are obtained in Sections 6 and 7. Finally, conclusions of the paper are provided in Section 8.

2. Failure times

A unit begins to operate at time 0 and will operate for an infinite time span. The unit undergoes minimal repairs [2, p. 96] at failures, where the time for each repair is supposed to be negligible. Let $0=S_0\le S_1\le\cdots\le S_{n-1}\le S_n\le\cdots$ be the successive random failure times and $X_n\equiv S_n-S_{n-1}$ $(n=1,2,\ldots)$ be the variable times between failures with distribution

$$\Pr\{X_n\le x\,|\,S_{n-1}=t\}=\frac{\overline{F}(t)-\overline{F}(t+x)}{\overline{F}(t)}=1-e^{-[H(t+x)-H(t)]}, \tag{1}$$

where failures occur at a non-homogeneous Poisson process with a mean value function $H(t)$ [16, p. 46; 14, p. 78], and $F(t)\equiv 1-e^{-H(t)}$ and $\overline{F}(t)\equiv 1-F(t)$ [11, p. 96].

Letting $N(t)$ be the number of failures in $[0,t]$, then the probability that failures occur k times in $[0,t]$ is

$$\Pr\{N(t)=k\}=\frac{[H(t)]^k}{k!}e^{-H(t)}\quad (k=0,1,2,\ldots), \tag{2}$$

$$E\{N(t)\}=\sum_{k=0}^{\infty}k\Pr\{N(t)=k\}=H(t). \tag{3}$$

From [11, p. 97], we obtain

$$E\{X_k\}=\int_0^\infty\frac{[H(t)]^{k-1}}{(k-1)!}e^{-H(t)}\,dt\quad (k=1,2,\ldots), \tag{4}$$

$$E\{S_n\}=\sum_{k=0}^{n-1}\int_0^\infty\frac{[H(t)]^k}{k!}e^{-H(t)}\,dt\quad (n=1,2,\ldots). \tag{5}$$

It is assumed that $H(t_n)=n$ and $x_n\equiv t_n-t_{n-1}$, where t_n $(n=1,2,\ldots)$ represents the time when the expected number of failures is n, then $H(x_n+t_{n-1})-H(t_{n-1})=1$ represents that the expected number of failures in $[t_{n-1},t_{n-1}+x_n]$ equals to 1. When the failure time of the unit has a Weibull distribution, i.e., $F(t)=1-e^{-t^m}$ and $H(t)=t^m$ $(m\ge 1)$, then (4) and (5) are

$$E\{X_k\}=\int_0^\infty\frac{t^{(k-1)m}}{(k-1)!}e^{-t^m}\,dt=\frac{1}{m}\frac{\Gamma(k-1+1/m)}{(k-1)!}\quad (k=1,2,\ldots), \tag{6}$$

$$E\{S_n\}=\sum_{k=1}^{n}E\{X_k\}=\frac{\Gamma(n+1/m)}{(n-1)!}\quad (n=1,2,\ldots). \tag{7}$$

Furthermore, when $(t_n)^m=n$, i.e., $t_n=n^{1/m}$, we obtain

$$x_k=t_k-t_{k-1}=k^{1/m}-(k-1)^{1/m}\quad (k=1,2,\ldots). \tag{8}$$

It is much easier to compute x_k in (8) than to compute $E\{X_k\}$ in (6). Table 1 presents exact $E\{X_k\}$ and approximate x_k when $H(t)=t^m$ for $m=1.5,2.0,3.0$. This shows that the approximate x_k is less than or equal to $E\{X_k\}$ when $k\ge 2$ and becomes very good approximation for the exact $E\{X_k\}$ as k becomes larger.

3. Age replacement

An operating unit has a failure distribution $F(t)$ and failure rate $h(t)\equiv f(t)/\overline{F}(t)$, where $f(t)$ is a density function of $F(t)$. Consider the standard age replacement policy in which the unit is replaced at a planned time $T(0<T<\infty)$ or at failure, whichever occurs first. Then, the expected cost rate is [2, p. 87; 11, p. 72]

$$C_1(T)=\frac{c_1F(T)+c_2\overline{F}(T)}{\int_0^T\overline{F}(t)\,dt}, \tag{9}$$

where c_1 and c_2 $(c_2<c_1)$ are respective replacement costs at failure and at time T. If the failure rate $h(t)$ increases strictly to ∞, then an optimal T^* minimizing $C_1(T)$ is given by a unique solution of the equation

$$h(T)\int_0^T\overline{F}(t)\,dt-F(T)=\frac{c_2}{c_1-c_2}. \tag{10}$$

From the above standard age replacement, we find that the only interest is to observe replacement actions that are done before the first failure or at the first failure. We suppose that

Table 1
Exact $E\{X_k\}$ and approximate x_k when $H(t)=t^m$.

k	$m=1.5$		$m=2.0$		$m=3.0$	
	$E\{X_k\}$	x_k	$E\{X_k\}$	x_k	$E\{X_k\}$	x_k
1	0.903	1.000	0.886	1.000	0.893	1.000
2	0.602	0.587	0.443	0.414	0.298	0.260
3	0.502	0.493	0.332	0.318	0.198	0.182
4	0.446	0.440	0.277	0.268	0.154	0.145
5	0.409	0.404	0.242	0.236	0.129	0.123
6	0.381	0.378	0.218	0.213	0.111	0.107
7	0.360	0.357	0.200	0.196	0.099	0.096
8	0.343	0.341	0.186	0.183	0.090	0.087
9	0.329	0.327	0.174	0.172	0.082	0.080
10	0.317	0.315	0.164	0.162	0.076	0.074
20	0.248	0.248	0.114	0.113	0.047	0.046
30	0.216	0.216	0.092	0.092	0.035	0.035
50	0.182	0.182	0.071	0.071	0.025	0.025