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Short communication

Incorrect modeling of the failure process of minimally repaired systems under random conditions: The effect on the maintenance costs

Gianpaolo Pulcini

Istituto Motori, CNR, Naples, Italy

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ABSTRACT

This note investigates the effect of the incorrect modeling of the failure process of minimally repaired systems that operates under random environmental conditions on the costs of a periodic replacement maintenance. The motivation of this paper is given by a recently published paper, where a wrong formulation of the expected cost for unit time under a periodic replacement policy is obtained. This wrong formulation is due to the incorrect assumption that the intensity function of minimally repaired systems that operate under random conditions has the same functional form as the failure rate of the first failure time. This produced an incorrect optimization of the replacement maintenance. Thus, in this note the conceptual differences between the intensity function and the failure rate of the first failure time are first highlighted. Then, the correct expressions of the expected cost and of the optimal replacement period are provided. Finally, a real application is used to measure how severe can be the economical consequences caused by the incorrect modeling of the failure process.

1. Introduction

In several situations, repairable systems operate under different environmental conditions that determine heterogeneity in the system failure probability. Some of the first papers on this topic are those of Bain and Wright [1] and Engelhardt and Bain [2], who studied the lack of fit of the Poisson model to the number of failures of minimally repaired systems that operate under different environmental conditions.

More recently, Sgarbossa et al. [3] dealt with the same problem, and used the "new" concept of "systemability" to obtain the reliability characteristics of repairable systems whose failure process is a non-homogeneous Poisson process with power-law intensity function (say, a Power Law process, PLP), where a multiplicative factor η is assumed to be random. Such a random factor represents the system operating environment that can change among the systems and hence affects differently the system failure probability.

In particular, Ref. [3] assumed that:

(a) the systems operate under different environmental conditions and are minimally repaired at failure,
(b) the first failure time T_1 is Weibull distributed, with failure rate $r(t;\eta)=\eta\lambda\gamma t^{\gamma-1}$, and
(c) the environment factor η is a gamma distributed random variable, whose probability density function (pdf) is $g(\eta)=\beta^{\alpha}\eta^{\alpha-1}\exp(-\beta\eta)/\Gamma(\alpha)$.

Under such assumptions, Sgarbossa et al. [3] derived the failure rate of T_1 under heterogeneous conditions, that they called the "Systemability failure rate function", by using the following relationship:

$$r_s(t)=-\frac{\partial R_s(t)}{\partial t}\cdot\frac{1}{R_s(t)}=\frac{\alpha\lambda\gamma t^{\gamma-1}}{\beta+\lambda t^{\gamma}},\tag{1}$$

where $R_s(t)$ is the "systemability function" obtained by averaging the Weibull reliability $R(t;\eta)=\exp\left[-\int_0^t r(z;\eta)\mathrm{d}z\right]$ over the distribution of η

$$R_s(t)=\int_0^{\infty}\exp\left[-\int_0^t r(z;\eta)\mathrm{d}z\right]g(\eta)\mathrm{d}\eta=\left[\frac{\beta}{\beta+\lambda t^{\gamma}}\right]^{\alpha}.\tag{2}$$

Note that the so-called "systemability function" (2) was derived almost half a century ago by Harris and Singpurwalla [4] and by Dubey [5] under a bit different parameterization. From (2), the probability density function (pdf) of T_1 resulted in

$$f_s(t)=-\frac{\partial R_s(t)}{\partial t}=\frac{\alpha\lambda\gamma\beta^{\alpha}t^{\gamma-1}}{(\beta+\lambda t^{\gamma})^{\alpha+1}}.\tag{3}$$

Sgarbossa et al. [3] assumed that the failure intensity $u_s(t)$ of the failure processes has the same functional form as the failure

E-mail address: g.pulcini@im.cnr.it

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Notations

t	Operating time
T_1	First failure time of the repairable system, a random variable
t_i	i-th failure time, $i = 1, 2, \ldots$, an observed value
η	Environment factor, a random variable
$g(\eta)$	Probability density function of η
$g(\eta; t)$	Conditional probability density function of η, given that $T_1 > t$
α, β	Shape, scale parameters of $g(\eta)$ and $g(\eta; t)$
$r(t; \eta)$	Conditional failure rate of T_1, given η
γ, λ	Shape, scale parameters of $r(t; \eta)$
$R_s(t)$	Systemability function
$r_s(t)$	Systemability failure rate function of T_1
$r_m(t)$	Mixture failure rate of T_1
$u_s(t)$	Systemability failure intensity
$\mu_m(t)$	Mixture failure intensity
μ_t	Stochastic failure intensity
$N(t)$	Number of failures up to t, a random variable
$N(t^-)$	Number of failures occurred immediately before t
$M_s(t)$	Mean number of failure up to t by using $u_s(t)$
$M_m(t)$	Mean number of failure up to t by using $\mu_m(t)$
t_p	Replacement period
c_p	Average preventive replacement cost
c_f	Average repair cost
$\mathrm{UEC}_{\alpha\beta}(t_p)$	Expected cost for unit time by using $u_s(t)$
$\mathrm{UEC}_m(t_p)$	Expected cost for unit time by using $\mu_m(t)$
t_p^*	Optimal replacement period

rate (1) of T_1, that is $u_s(t) = r_s(t) = \alpha\lambda\gamma t^{\gamma-1}/(\beta+\lambda t^\gamma)$. Thus, they evaluated the mean number of system failures up to time t by integrating the "Systemability intensity function" $u_s(t)$

$$M_s(t) = \int_0^t u_s(z)\mathrm{d}z = \int_0^t \frac{\alpha\lambda\gamma z^{\gamma-1}}{\beta+\lambda z^\gamma}\mathrm{d}z = \alpha \ln\left(1+\frac{\lambda}{\beta}t^\gamma\right). \tag{4}$$

By using (4), they obtained the expected cost for unit time (UEC) under a periodic replacement policy

$$\mathrm{UEC}_{\alpha\beta}(t_p) = \frac{c_p + c_f M_s(t_p)}{t_p} = \frac{c_p + c_f \alpha \ln(1+\lambda t_p^\gamma/\beta)}{t_p}, \tag{5}$$

where t_p is the replacement period, and c_p and c_f are the average preventive replacement cost and the average repair cost, respectively. Minimizing (5), Sgarbossa et al. [3] derived their optimal replacement period.

However, even when the systems are minimally repaired, the assumption that the intensity function of the failure process has the same functional form as the failure rate of the first failure time holds only when the operating environments are homogeneous. Indeed, in the presence of heterogeneous conditions, the failure intensity differs, both conceptually and numerically, from the failure rate $r_s(t)$ of Eq. (1), and then the assumption that $u_s(t) = r_s(t)$ is incorrect. As a consequence, both the expected cost for unit time $\mathrm{UEC}_{\alpha\beta}(t_p)$ and the optimal replacement period provided in [3] are wrong. Evidence of such errors is furnished by the unreasonable behavior of the UEC (5), that was not adequately investigated in [3].

In this note, the conceptual and mathematical differences between the failure rate of the first failure time and the intensity function of repairable systems operating under heterogeneous environmental conditions are first discussed. The correct formulation of the intensity function is then given, from which the correct modeling of the failure process is derived. The correct expressions of the expected cost for unit time and of the optimal replacement period are also derived. Finally, the real case study illustrated in [3] is here reanalyzed in order to measure the effects on the maintenance costs of the incorrect modeling of the failure process.

2. The intensity function and the mean number of system failures under heterogeneous conditions

In presence of heterogeneity in the operating environments, the failure rate of the first failure time T_1 of a repairable system, say $r(t; \eta)$, is a conditional probability that is affected by the fact that the probability distribution of the environment factor η (that, in general, can be a vector of random variables) of the systems that have not yet experience their first failure at time t changes with t, and hence differs from the (initial) distribution $g(\eta)$ at time $t = 0$. This occurs because the systems operating under more stressful conditions are more likely to experience early their first failure, and hence at large times t only systems operating under less stressful conditions have never yet failed. Thus, as shown in Finkelstein [6–8], the mixture failure rate $r_m(t)$ is given by

$$r_m(t) = \int_0^\infty r(t;\eta)g(\eta;t)\mathrm{d}\eta = \frac{\int_0^\infty f(t;\eta)g(\eta;t)\mathrm{d}\eta}{\int_0^\infty R(t;\eta)g(\eta)\mathrm{d}\eta}, \tag{6}$$

where $f(t; \eta)$ is the conditional pdf of the first failure time of a system operating, given the environment η, and $g(\eta; t)$ is the (conditional) pdf of the environments η under which operate the systems that have not yet experienced their first failure at time t, that is, the systems for which $T_1 > t$. As shown in [6–8], $g(\eta; t)$ is given by

$$g(\eta;t) = \frac{g(\eta)R(t;\eta)}{\int_0^\infty R(t;\eta)g(\eta)\mathrm{d}\eta}, \quad \eta \ge 0. \tag{7}$$

When the first failure time is Weibull distributed, the environment factor η is a scalar variable, gamma distributed, that acts multiplicatively on the failure rate (see assumptions (a)–(c)), the (conditional) pdf of η is given by

$$g(\eta;t) = \frac{(\beta+\lambda t^\gamma)^\alpha \eta^{\alpha-1}}{\Gamma(\alpha)} \exp[-\eta(\beta+\gamma t^\gamma)], \tag{8}$$

that is still a gamma distribution with (time independent) shape parameter α and (time dependent) scale parameter $\beta+\lambda t^\gamma$. Its (conditional) expectation, say $E\{\eta; t\} = \alpha/(\beta+\lambda t^\gamma)$, decreases with t, irrespectively from the value of γ.

Fig. 1 shows the (conditional) pdf $g(\eta; t)$ for different values of the operating time t, evaluated by using the parameters values used in [3, Example 4.2]: $\lambda = 0.005$, $\gamma = 3$, $\alpha = 5$, and $\beta = 25$. The conditional expectation $E\{\eta; t\}$ is also plotted. It is quite evident that, as t increases, the operating environments of the systems that have not yet experienced their first failure are those less stressful.

From (6) and (8), we have that the mixture failure rate, under the assumptions (a)–(c), is given by

$$r_m(t) = \frac{\alpha\lambda\gamma t^{\gamma-1}}{\beta+\lambda t^\gamma}, \tag{9}$$

and hence the "systemability failure rate function" $r_s(t)$ of Eq. (1) is no more than the mixture failure rate under heterogeneous conditions proposed, 20 years ago, by Gurland and Sethuraman [9] (see also the more recent book of Finkelstein [7]). When the individual failure rates $r(t; \eta)$ are constant or decreasing, that is,