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Modelling repairable systems with an early life under competing risks and asymmetric virtual age

Isha Dewan [a], Yann Dijoux [b,*]

[a] Indian Statistical Institute, New Delhi, India
[b] Université de Technologie de Troyes, 12 rue Marie Curie, CS 42060, 10004 Troyes Cedex, France

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ABSTRACT

In this paper, complex repairable systems presenting infant failures are considered. Different maintenance activities can be carried out such as corrective maintenances and planned preventive maintenances. The maintenance process is described using the competing risks framework under imperfect maintenance. Asymmetric virtual age models are assumed to characterize the maintenance efficiency. Statistical inference procedures are developed considering whether or not the causes of failure are recorded. Properties of the model are presented along with numerical estimations and an application to real data.

1. Introduction

Multiple maintenance activities are carried out on repairable industrial systems throughout their service life. A corrective maintenance (CM) is performed after the occurrence of a failure and intends to bring the system back to operational status for the longest time possible. Two failure modes are considered: the system fails either due to internal defects (manufacturing, design, assembly or installation-induced defects, operator's misuse of a new equipment) or its general ageing. A preventive maintenance (PM) is performed while the system is functional and intends to prolong its life. Planned preventive maintenance actions are carried out at predetermined times. Maintenance efficiencies are traditionally assumed to be either perfect or minimal. Intermediate configurations have been characterized with imperfect maintenance models [24]. The virtual age models introduced in [22] are the most well-spread of these models. A vast majority of the virtual age models consider wearing-out systems with increasing failure rate. To take into account complex systems with infant failures, bathtub curves are commonly employed [27,28,30]. Virtual age models have been adapted to bathtub shaped intensities in [10,11].

Bathtub shaped functions are the aggregation of different phenomena: a decreasing portion corresponding to failures caused by internal defects, a potential constant portion associated with random failures and an increasing portion corresponding to failures due to ageing. In the following, the random failure mode is not considered but the modelling could be extended consequently. In the paper, we propose to dissociate the early failure mode from the ageing failure mode. To that purpose, the competing risks framework [9] is used to describe a risk due to the ageing and another due to internal defects. Preventive maintenances are assumed to be planned [29], then the PM times correspond to censoring times of the CM process. As the nature of the risks are different, it is natural to assume that both risks are independent. Moreover, it seems realistic that the actions performed and their efficiency differ depending on the kind of maintenance. Different virtual age assumptions have to be considered for each risk. This situation has been developed in the context of perfect or minimal repairs in [2] and the concept has been extended to the notion of asymmetric virtual ages as defined in [16]. It seems reasonable to assume a minimal repair for the risk of internal failure and a classical virtual age model for the risk related to the ageing and preventive maintenances. If failure modes are recorded, the classical results of independent competing risks models associated to imperfect maintenance can easily be derived. Unfortunately, the nature of a failure is not always observed or recorded and in practice, this information is rarely provided in data sets. In this context, an algorithm is developed to estimate the parameters of the model under missing observations. This algorithm is applied considering two data sets: the first one only consists of corrective maintenances and the second one consists of PM and CM. For both real data, the nature of the failures is unknown.

The paper is organized as follows. Imperfect maintenance models under competing risks and virtual age are introduced in

* Corresponding author. Tel.: +33 3 25 71 58 04.
E-mail addresses: isha@isid.ac.in (I. Dewan), yann.dijoux@utt.fr (Y. Dijoux).

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Section 2. In Section 3, a competing risks model with asymmetric virtual age is proposed for systems with an early life. Estimation procedures are developed in Section 4 and are applied to real data.

2. The maintenance process

2.1. Notations

Maintenance times and types are recorded throughout the life of a repairable system. Maintenance durations are assumed to be negligible or not taken into account. In the following, the notations are introduced:

- $\{C_k\}_{k\geq 1}$ are the maintenance times (CM and PM), with $C_0=0$.
- $\{W_k\}_{k\geq 1}$ are the times between maintenances, $W_k=C_k-C_{k-1}$.
- $K=\{K_t\}_{t\geq 0}$ is the counting maintenance (CM and PM) process. The counting process is assumed to be simple.
- $\{U_k\}_{k\geq 1}$ are the indicators of maintenance types:

$$U_k=\begin{cases}1 & \text{if the } k\text{th maintenance is preventive (PM)}\\ 0 & \text{if the } k\text{th maintenance is corrective, due to the ageing}\\ -1 & \text{if the } k\text{th maintenance is corrective, due to an internal defect}\end{cases} \quad (1)$$

The maintenance process is completely given by three type-specific intensities [1,23] defined as

$$\forall i\in\{-1,0,1\},\quad \forall t\geq 0,$$
$$\lambda_t^i=\lim_{\Delta t\to 0}\frac{1}{\Delta t}P(K_{t+\Delta t}-K_{t^-}=1, U_{K_{t^-}+1}=i|\mathcal{H}_{t^-}) \quad (2)$$

where $\mathcal{H}_{t^-}$ is the history of the maintenance process at time t. Without external variable, it corresponds to the times and kinds of maintenances performed before t. Moreover, if PM are planned, it is not necessary to define the intensity λ_t^1, as the PM process can be seen as a deterministic censoring of the corrective maintenance process. An example of a trajectory of the maintenance process is presented in Fig. 1. In the following, bold characters denote vectors as in $\mathbf{W_k}=(W_1,\ldots,W_k)$.

2.2. The competing risks framework

The maintenance process is developed in the competing risks framework [7], adapted to characterize an event which can be observed from the realization of one out of several processes running simultaneously. After the kth maintenance, the nature of the next maintenance is undetermined. Let Y_{k+1} be the latent time to next PM, Z_{k+1} be the latent time to next ageing failure and R_{k+1} be the latent time to next internal failure. The actual observations are the time to next maintenance $W_{k+1}=\min(Y_{k+1},Z_{k+1},R_{k+1})$ and the maintenance indicator U_{k+1}. U_{k+1} equals to 1 if a PM is carried out, 0 if the maintenance is corrective and due to the ageing (ageing failure) and -1 if it is a CM due to internal defects (internal failure).

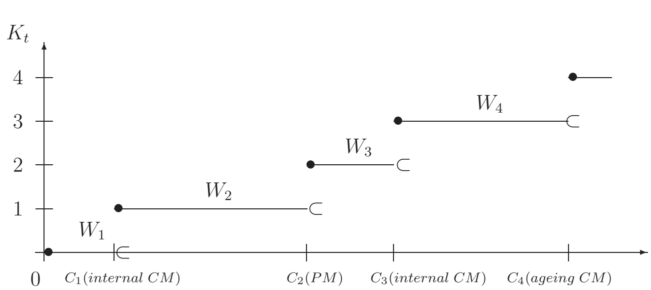

Fig. 1. Example of trajectory of the maintenance process.

Classical competing risks [9] assume that the triplets $\{(Y_k,Z_k,R_k)\}_{k\geq 1}$ are independent and identically distributed. It implies that the effect of each maintenance is to restore the system as good as new (AGAN). In this configuration, to characterize the model, it is sufficient to consider the three-dimensional survival function:

$$\mathcal{S}_1(y,z,r)=P(Y_1>y,Z_1>z,R_1>r) \quad (3)$$

In many situations, the initial risks are assumed to be independent, which imply that the survival function (3) is the product of three marginal survival functions S_Y, S_Z, and S_R. These functions are the survival functions of Y, Z and R, respectively. If PM are planned, Y_1 corresponds to a deterministic censoring of the two other variable risks Z_1 and R_1. Hence it is sufficient to define the two-dimensional survival function as in the following equation:

$$S_1(z,r)=P(Z_1>z,R_1>r) \quad (4)$$

In practice, maintenances reduce the failure intensity but do not restore the system As Good As New. Imperfect maintenance [24] has then been introduced and has been adapted to competing risks in [16] for condition-based preventive maintenance and corrective maintenance. Our context is different as PM can be planned and multiple kinds of CM are considered, but the principle is identical. It assumes that the triplets $\{(Y_k,Z_k,R_k)\}_{k\geq 1}$ are not independent and identically distributed. The couples $\{(W_k,U_k)\}_{k\geq 1}$ are therefore also not iid and the effect of each maintenance can be modelled as imperfect. The usual competing risks functions are naturally generalized by introducing a conditioning on the history of the maintenance process. Then, as shown in [16], the intensities and likelihood function can be expressed in terms of the conditional generalized survival functions $\mathcal{S}_{k+1}$ defined in (5). If PM are assumed to be planned, the conditional generalized survival functions S_{k+1} can be expressed as in (6) and the PM risks $\{Y_k\}_{k\geq 1}$ can be modelled as deterministic censoring times:

$$\mathcal{S}_{k+1}(y,z,r;\ \mathbf{W_k},\mathbf{U_k})=P(Y_{k+1}>y,Z_{k+1}>z,R_{k+1}>r\mid \mathbf{W_k},\mathbf{U_k}) \quad (5)$$

$$S_{k+1}(z,r;\ \mathbf{W_k},\mathbf{U_k})=P(Z_{k+1}>z,R_{k+1}>r\mid \mathbf{W_k},\mathbf{U_k}) \quad (6)$$

2.3. Asymmetric virtual age models

The conditioning of $(\mathbf{W_k},\mathbf{U_k})$ in (5) and (6) allows us to define the impact of the previous maintenances by using imperfect maintenance assumptions. Virtual age models introduced in [22] and developed in [18,19] are the most widespread imperfect maintenance models. They assume that after the kth maintenance, the system behaves as a new one having survived until an effective age A_k. This is a positive random variable which is commonly a function of the previous times and kinds of maintenance. Considering only one kind of maintenance, the basic assumptions on the effective ages are as follows:

- $A_k=C_k$ and maintenances are minimal or As Bad As Old (ABAO).
- $A_k=0$ and maintenances are perfect and restore the system to As Good As New (AGAN).
- The Brown–Proschan (BP, [4]) model assumes that each maintenance is AGAN with probability p and ABAO with probability $1-p$, where p reflects the maintenance efficiency. After a maintenance, the effective age of the system is the time elapsed since the last perfect maintenance.
- The Arithmetic Reduction of Age model with memory 1 (ARA_1, [15]) assumes that after a maintenance, the effective age is proportional to the real age of the system $A_i=(1-\rho)C_i$, where $\rho\in[0,1]$ reflects the maintenance efficiency.