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Weibull and lognormal Taguchi analysis using multiple linear regression

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ABSTRACT

The paper provides to reliability practitioners with a method (1) to estimate the robust Weibull family when the Taguchi method (TM) is applied, (2) to estimate the normal operational Weibull family in an accelerated life testing (ALT) analysis to give confidence to the extrapolation and (3) to perform the ANOVA analysis to both the robust and the normal operational Weibull family. On the other hand, because the Weibull distribution neither has the normal additive property nor has a direct relationship with the normal parameters (μ, σ), in this paper, the issues of estimating a Weibull family by using a design of experiment (DOE) are first addressed by using an L_9 (3^4) orthogonal array (OA) in both the TM and in the Weibull proportional hazard model approach (WPHM). Then, by using the Weibull/Gumbel and the lognormal/normal relationships and multiple linear regression, the direct relationships between the Weibull and the lifetime parameters are derived and used to formulate the proposed method. Moreover, since the derived direct relationships always hold, the method is generalized to the lognormal and ALT analysis. Finally, the method's efficiency is shown through its application to the used OA and to a set of ALT data.

1. Introduction

In failure time regression models, as is the case of the Weibull proportional hazard model (WPHM) and the accelerated life testing (ALT) models, e.g. Arrhenius, power law and Eyring models, the covariates (X_1, X_2, …, X_k) (such as temperature and voltage) are taken to be constant over time and measured by interval [1]. Thus, data are collected by using a design of experiment (DOE). On the other hand, although the lifetime data are collected through the time, because of the aging and the wear out affects, the lifetime data is random. Thus, it must be modeled by a probability density function (pdf). Moreover, because in the DOE each row represents a different way to run the process, each row (as in Figs. 1 and 2) presents its own Weibull family. Thus, because into the DOE families the shape parameters β_i present different values, the Weibull closure property does not hold (this property means that β must be constant in all the rows) [2], and as a consequence, the Taguchi method (TM) could not be applied directly to analyze Weibull data. On the other hand, with the objective to use the TM to analyze Weibull data, recent research has been done. Among others, we found [3–8]. However, although [6–8] used the relations of the Weibull parameters with those of the smallest extreme value distribution (Gumbel distribution) in their analysis, because they neither identify the fact that the DOEs presents different Weibull families in their rows nor the direct relationships of the Weibull and lifetime parameters, their methods are not as efficient as our proposed method does.

On the other hand, in this paper by using both the direct relations between the Weibull and Gumbel (Type I-extreme value distribution), and the fact that the TM uses a linear polynomial to predict the signal to noise ratio (S/N), the statistical analysis to use directly the TM to analyze Weibull data is presented. Since in the proposed method; (1) the Weibull scale parameter η_i is directly related with the mean of the logarithm (μ_{xi}) of the DOE data, (2) the Weibull shape parameter β_i is directly related with the standard deviation of the logarithm of the DOE data (σ_{xi}), and (3) because the logarithm behavior is asymptotically normal [9], then, based on the relations between the Gumbel and the lognormal distribution, the findings were also generalized to the lognormal distribution. The main contribution of the paper is that it lets practitioners (1) to determine by using the TM, the Weibull family of the robust setting (or any other desired setting), (2) to give confidence to the extrapolation in any ALT analysis by estimating the corresponding operational Weibull family and then,

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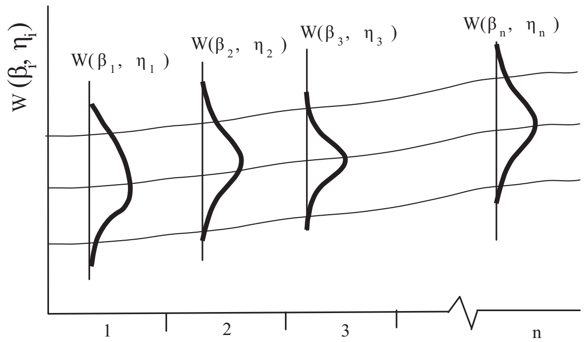

Fig. 1. Orthogonal array levels.

Run number	Inner array				Outer array Z_1 / Z_2	1 / 1	1 / 2	2 / 1	2 / 2
	x_1	x_2	x_3	x_4					
1	1	1	1	1					
2	1	2	2	2					
3	1	3	3	3					
4	2	1	2	3	(Life time data)				
5	2	2	3	1					
6	2	3	1	2					
7	3	1	3	2					
8	3	2	1	3					
9	3	3	2	1					

Fig. 2. Orthogonal array L(9) 3^4.

basing on this Weibull family, to incorporate its expected lifetimes to the ALT analysis and (3) to perform the ANOVA analysis to both, the robust TM and the normal operational Weibull families.

On the other hand, in order to show how the method works two sets of data are used. The Besseris'data [10] is used to show how to determine the robust Weibull family by using the TM analysis, and the Vassiliou and Meta's data [11] is used to give confidence to the extrapolation in ALT analysis by estimating the normal operational Weibull family and by incorporating its expected times to the ALT analysis. The structure of the paper is as follows; Section 2 addresses the problems of analyze Weibull data in both the TM and the WPHM approach. Section 3, presents the proposed method. In Section 4 the generalization of the proposed method to the ALT analysis is given. Finally, the paper ends in Section 5 with the conclusions.

2. Problem statement

The fact that because in a DOE each row presents its own Weibull family, then neither the TM nor the WPHM could be used to analyze Weibull data is presented in the following sections.

2.1. Taguchi method

Robust design is Dr. Taguchi's approach used to determine the optimum configuration of the design parameters for performance, quality and cost. It provides a systematic and efficient approach to find the near optimum combination of the design parameters in such that the product is functional, exhibits a high level of performance and is robust to noise factors [12]. The three major steps in robust design by applying the TM are system design, parameter design, and tolerance design [13]. In the TM, the factors are arranged in special tabular tables provided by Dr. Taguchi and known as orthogonal array (OA) [14], (see Fig. 2). On the other hand, by performing the statistical analysis in the TM, we determine the level setting of the parameters for which the performance characteristic is robust. This statistical analysis is performed based on a function called signal to noise ratio (S/N). The S/Ns were formulated based on the normal distribution, and they take into account both the mean (μ) and the standard deviation (σ). Moreover, the S/N formulation is such that, regardless the type of criterion is, the S/N transformation is always interpreted in the same way, the larger the S/N the better [15]. However, their formulation depends on the criteria of the quality characteristic to be optimized (for dynamic criteria see [16]). The standard non dynamic criteria are: smaller the better quality characteristic given by

$$S/N = -10\log\left(\frac{1}{n}\right)\sum_{i=1}^{n} y_i^2 \tag{1}$$

nominal the best quality characteristic given by

$$S/N = 10\log\left(\frac{\hat{\mu}^2}{\hat{\sigma}^2}\right) \tag{2}$$

and larger the better quality characteristic given by

$$S/N = -+10\log\left(\frac{1}{n}\right)\sum_{i=1}^{n}\frac{1}{y_i^2} \tag{3}$$

In (1) and (3), n is the number of replicates of the OA data, and y_i is the corresponding response value. In (2), μ and σ are the mean and the standard deviation of the normal distribution given by

$$f(t) = \frac{1}{\sigma_t\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{t-\mu}{\sigma_t}\right)^2\right) \tag{4}$$

On the other hand, from a set of n data, μ is given by

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{5}$$

Moreover, observe that (5) in terms of the OA covariates could be estimated as

$$\hat{\mu}_j = b_0 + \beta_j X_j \tag{6}$$

where b_0 represents the ordinate to the origin and β_j is the corresponding slope (effect) of the X_j covariate. In the same way, the variance is estimated as

$$\hat{\sigma}_{tj}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(y_{ij} - \hat{\mu}_j\right)^2 \tag{7}$$

On the other hand, based on the linear relationship defined in (6), the prediction polynomial model in the TM is given by

$$\hat{T} = \sum_{i=1}^{n} X_{ij} - (n-1)\overline{\mu} \tag{8}$$

In (8) X_{ij} represents the jth-covariate, and i represents its ith-level, e.g. if the jth-covariable is the temperature and it has three levels, then $i=(T_1, T_2, T_3)$, and $\overline{\mu}$ is the overall mean. Here, it is important to note that by using (8), no matter which S/N we use, *the estimated* $\hat{\mu}$ *and* $\hat{\sigma}$ *are always the same* and they are the parameters of the normal distribution defined in (4). It is to say, the *TM works efficiently when the analyzed data fits to a normal distribution*. On the other hand, because there is not a direct relationship between the Weibull and the normal parameters, when the analyzed data follows a Weibull distribution, the TM analysis could not be directly applied. To see that, let us use the data published in [10]. In this data, the analyzed factors were aluminum-alloy content; in manganese (Mn) and magnesium (Mg) as well as hot mill pass counts (HMPC) and cold mill reduction rate (CMR). Mn, Mg, and CMR data were measured in percentage units, and in order to quantify the reliability performance, the time duration was monitored in seconds until the can-bottom