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# Adaptive finite difference solutions of Liouville equations in computational uncertainty quantification

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**Abstract**

In the context of computational uncertainty quantification, we present a finite difference based solution of a Liouville equation that describes the evolution of a conditional probability density function depending on state variables and model input parameters. The efficiency of the numerical solution is enhanced through (i) a quadrature-based sampling of random variables corresponding to model input parameters and (ii) time-adaptive methods for determining the computational grid in the space of state or response variables. The proposed method, which allows for accurate and computationally efficient determination of the conditional density and resulting statistical moments, is applied to a decay problem and a problem which contains nonlinear deterministic dynamics with multiple equilibrium points. Through comparison with exact solutions and direct numerical simulation on a fixed time-invariant grid, it is demonstrated that the proposed methodology not only is able to accurately predict the large time behavior of the response density but also can significantly reduce the computational costs.

*Keywords:* finite difference solution, computational uncertainty quantification, Liouville equation



## 1. Introduction

Uncertainty quantification (UQ) generally involves determination of the effects of uncertainty in system input parameters on system output vari-

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