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Reliability demonstration methodology for products with Gamma Process by optimal accelerated degradation testing

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ABSTRACT

For products with high reliability and long lifetime, accelerated degradation testing (ADT) may be adopted during product development phase to verify whether its reliability satisfies the predetermined level within feasible test duration. The actual degradation from engineering is usually a strictly monotonic process, such as fatigue crack growth, wear, and erosion. However, the method for reliability demonstration by ADT with monotonic degradation process has not been investigated so far. This paper proposes a reliability demonstration methodology by ADT for this kind of product. We first apply Gamma process to describe the monotonic degradation. Next, we present a reliability demonstration method by converting the required reliability level into allowable cumulative degradation in ADT and comparing the actual accumulative degradation with the allowable level. Further, we suggest an analytical optimal ADT design method for more efficient reliability demonstration by minimizing the asymptotic variance of decision variable in reliability demonstration under the constraints of sample size, test duration, test cost, and predetermined decision risks. The method is validated and illustrated with example on reliability demonstration of alloy product, and is applied to demonstrate the wear reliability within long service duration of spherical plain bearing in the end.

1. Introduction

Reliability demonstration test is commonly adopted in product development phase to validate whether its reliability meets the specified level. Accelerated life test (ALT) could reduce test duration and has been applied in reliability demonstration for highly reliable products with long lifetime [1,2].

For many engineering circumstances, the reliability of highly reliable product is usually dominated by its performance degradation process. Accelerated degradation test (ADT) is consequently promoted as a feasible solution to reliability demonstration for this kind of products because it reduces test duration further by comparison with ALT. Yang [3] presented a reliability demonstration method based on degradation test under normal stress level using linear degradation model, which established the basis of demonstrating reliability by performance degradation. Huang [4] proposed an optimal ADT plan for reliability demonstration based on linear degradation model.

Compared with linear degradation model, stochastic process model can reasonably identify the random variation in degradation process. Baussaron [5] proposed a reliability demonstration method based on ADT using Wiener process model. Generally speaking, the performance degradation of product is usually a monotonic process, such as fatigue crack growth, wear, and erosion. Gamma process model is a strictly monotonic process model, and it is more reasonable for these occasions than Wiener process model [6]. Unfortunately, few literatures can be identified to deal with reliability demonstration method by ADT using the Gamma process model.

This paper proposes a reliability demonstration methodology for products with Gamma degradation process by ADT. The rest of this paper is organized as follows. Section 2 gives a motivating example. Section 3 delivers the models for the problem. Section 4 suggests a new reliability demonstration method by converting the required reliability level into allowable cumulative degradation in ADT and comparing the actual accumulative degradation with the allowable level. Section 5 presents an analytical optimal ADT design method for more efficient reliability demonstration. Section 6 offers the results that apply the proposed method to the motivating example. Section 7 applies the method to demonstrate the wear reliability within long service duration of spherical plain bearing. Section 8 concludes the paper.

2. Motivating example

The metal component subject to cyclic load will rupture when the length of fatigue crack reaches the threshold, which accounts

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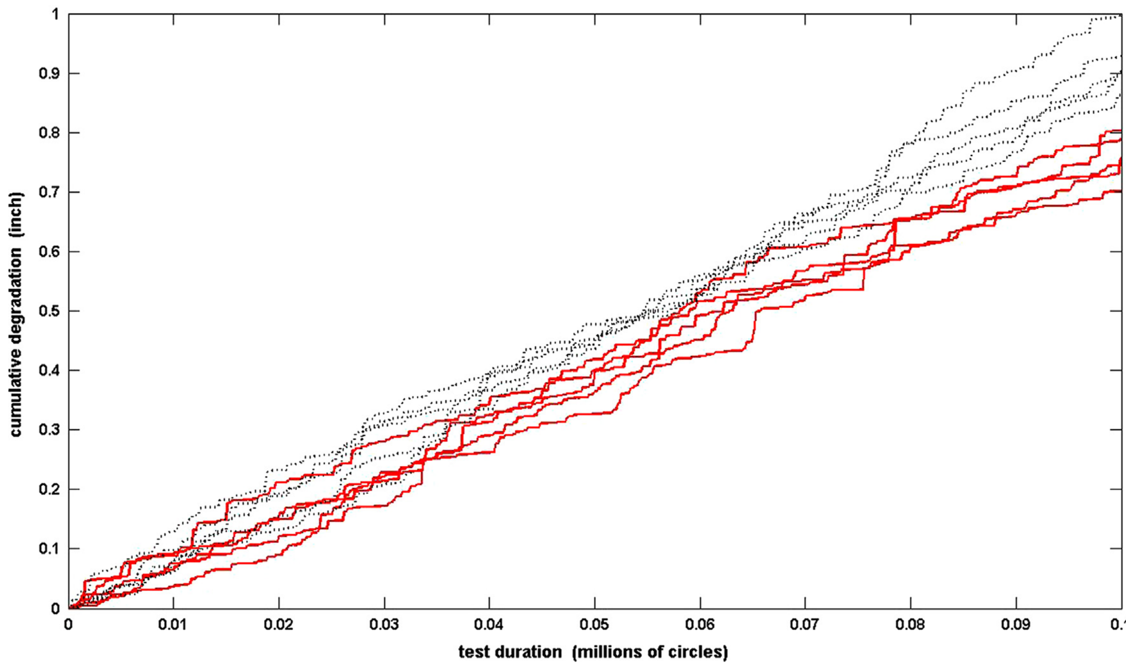

Fig. 1. Typical fatigue crack development process of an alloy product from ADT. (The solid lines denote the cumulative crack length of 5 products under an elevated stress level, and the broken lines denote a higher stress level.)

for a majority of failures for metal components. Meeker and Escobar [7] described such a problem. Fatigue crack development process of a kind of alloy product is shown in Fig. 1, which is a strictly monotonic process and can be described properly by Gamma process.

Suppose that the reliability of the alloy product required by consumer is $R(\beta) = 0.9016$ with risk $\beta = 0.10$ at $t_0 = 0.1286$ millions of circles, and that the acceptable reliability for producer is required to be $R(\alpha) = 0.9832$ with risk $\alpha = 0.30$ at the same time. If such a high reliability with long service duration is demonstrated by traditional fatigue life test, it will be unfeasible for the too lengthy test duration. So the reliability demonstration methodology by ADT for such kind of products with Gamma process should be investigated.

For this problem, there exist some issues worthy of further consideration:

- If ADT is employed, how can we judge whether the reliability satisfies the required level according to the degradation data acquired in ADT?
- Can we possibly optimize the test plan for efficient reliability demonstration, which means better demonstration confidence with the constraints on sample size, test duration, and test cost?

3. Models

3.1. Degradation model

Gamma process $\{X(t) \sim Ga(\nu t, u), t \geq 0\}$ with shape parameter ν and scale parameter u is a strictly monotonic degradation process with s-independent and non-negative increments $\Delta X(t) = X(t+\Delta t) - X(t), \Delta t > 0$, and its probability density function is defined as

$$f(\Delta x) = \frac{1}{\Gamma(\nu \Delta t) u^{\nu \Delta t}} \Delta x^{\nu \Delta t - 1} e^{-\Delta x / u} \tag{1}$$

where, $\Gamma(\nu \Delta t) = \int_0^\infty x^{\nu \Delta t - 1} e^{-x} \, dx$.

Suppose a failure occurs when the degradation reaches the specified threshold ϖ, then failure time T can be defined as the time when the degradation path cross ϖ, and the reliability at t can be expressed as

$$R(t) = P(T > t) = P(X(t) < \varpi) = Ga(\varpi | \nu t, u) \tag{2}$$

Park and Padgett derived the probability density function of T at t, but it is too complex to be used in practice [1,8]. They recommended a two-parameter Birnbaum–Saunders distribution be used for approximation

$$F(t) = P(T \leq t) = P(X(t) \geq \varpi) = \Phi\left(\sqrt{\nu t} - \sqrt{\frac{\varpi^2}{u^2 \nu t}}\right) \tag{3}$$

where $\Phi(\bullet)$ is standard s-normal cumulative distribution function, u and ϖ are s-independent from stress levels.

According to Central Limit Theorem, the approximation error will be small when t is large enough. As an illustration, the approximation error for the cumulative distribution of T with parameters employed in the following chapter is shown in Fig. 2.

3.2. Acceleration model

Refer to (3), the reliability for a given Gamma degradation process with u and ϖ is determined by νt in ADT for products with Gamma degradation process. The shape parameter ν reflects the effect of stress level S_k on the performance, and it determines the degradation rate of Gamma process. The scale parameter u reflects the random variation of the performance caused by variation of material, manufacturing, and environment, and it is s-independent from stress levels.

Acceleration model in ADT for products with Gamma degradation process is commonly assumed as [6,9,10]

$$\ln \nu_k = a + b \varphi(S_k) \tag{4}$$

where a and b are unknown coefficients, and $\varphi(S_k)$ is a function of S_k. If $\varphi(S_k) = \ln S_k$, acceleration model is inverse power law. If $\varphi(S_k) = 1/S_k$, it is the Arrhenius equation.