Reliability Engineering and System Safety 142 (2015) 472–481

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

An efficient algorithm for the multi-state two separate minimal paths reliability problem with budget constraint

Majid Forghani-elahabad [a,*], Nezam Mahdavi-Amiri [b]

[a] Universidade Federal do ABC – UFABC, Santo André, SP, Brazil
[b] Faculty of Mathematical Sciences, Sharif University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 4 January 2015
Received in revised form
4 May 2015
Accepted 16 June 2015
Available online 24 June 2015

Keywords:
Stochastic quickest path problem
Transmission time
Budget constraint
Minimal paths (MPs)
(d,T,b,P_1,P_2)-MP

ABSTRACT

Several researchers have worked on transmitting a given amount of flow through a network flow within fastest possible time, allowing flow to be transmitted through one or more paths. Extending this problem to the system reliability problem, the quickest path reliability problem has been introduced. The problem evaluates the probability of transmitting some given amount of flow from a source node to a sink node through a single minimal path in a stochastic-flow network within some specified units of time. Later, the problem has been extended to allow flow to be transmitted through two or more separate minimal paths (SMPs). Here, we consider the problem of sending flow through two SMPs with budget constraint. Presenting some new results, an efficient algorithm is proposed to solve the problem. The algorithm is illustrated through a benchmark ARPANET example. Computing complexity results, the algorithm is shown to be significantly more efficient than the existing ones. We also state how the optimal two SMPs with the best system reliability can be determined based on our proposed algorithm. Finally, testing on more than 10 000 generated random test problems, the practical efficiency of our algorithm is demonstrated in comparison with a recently proposed algorithm.

1. Introduction

After World War I, reliability was measured as the number of accidents per hour of flight time for one-, two-, and four-engine airplanes [1]. Nowadays, network reliability theory has extensively been applied to a variety of real-world systems such as transportation [2], mobile ad hoc wireless [3,4], power transmission and distribution [5,6], grid computing [7], manufacturing [8–10], and computer and communication [9,11]. Moreover, in some optimization problems such as maximizing system reliability [6,8] or optimal design of a network subject to reliability constraint [12], there is an increasingly significant need for efficiently computing or estimating the system reliability. Thus, the system reliability problem turns to be an important challenging problem for system engineers. For example, to construct a manufacturing system, Lin and Chang [10] exhibited each machine in the system by an arc and each inspection station by a node, and evaluated its reliability to produce d units of products with reworking actions, where d is a given demand level of product. In this kind of a network, the flow conservation law is not necessarily satisfied, i.e., the input and output are not necessarily equal [8–10]. Applying approximate methods such as Monte-Carlo simulation [4,13], or exact ones such as inclusion–exclusion principle [14,15], or sum of disjoint product [16,17], system reliability can be computed in terms of minimal cuts, or minimal paths [1–30].However, evaluating the system reliability is an NP-hard problem [30], and thus the problem continues to be interesting to investigate.

In a logical topology of an optical network, each node exhibits a destination, source (typically computers), or even an optical router, and each arc denotes a light path. Since data can be carried out on different wavelengths simultaneously, optical networks can be considered as a stochastic-flow network (SFN) [31,32]. In an SFN, system reliability can be considered as the probability of transmitting a given demand amount, d units, of data (flow) from a source node to a sink node [1–30]. A number of different algorithms have been proposed to compute this kind of reliability parameter [3,13–21]. For example, Yan and Qian [19] presented several new results along with an improved algorithm. Forghani-elahabad and Mahdavi-Amiri [21] proposed a new data structure and established some new results to present a new efficient algorithm. However, in an optical network, there are some time thresholds and when the transmission time exceeds the threshold, the transmission will be canceled, and so the transmission time is an important issue to be considered. Attending to transmission time, several studies have been made in terms of minimal paths

* Corresponding author. Tel.: +55 11 9 7756 4058.
E-mail addresses: m.forghani@ufabc.edu.br (M. Forghani-elahabad), nezamm@sina.sharif.edu (N. Mahdavi-Amiri).

http://dx.doi.org/10.1016/j.ress.2015.06.012
0951-8320/

according to the quickest path problem [23–27,33–38]. Quickest path problem is a version of shortest path problem in a network flow with arcs having two attributes, capacity and lead time [23–27,33–38].Although the inception of the problem may be attributed to the work by Moore [23], however, the notion of quickest path problem in our context has been introduced by Chen and Chin [33]. The problem is finding a path to transmit d units of flow from a source node to a sink node within a minimal transmission time [23–27,33–38]. Mentioning and rectifying two existing disadvantages of the previous works, Park et al. [34] proposed a label-setting algorithm to recognize the quickest path. Calvete et al. [35] considered the proposed algorithms for the quickest path problem without and with lower bound reliability constraints. Although the time complexity of their algorithm was the same as the one for the algorithm in [34], they showed their algorithm to be better than the proposed ones in [33,34] through numerical experiments. Noticing the two disadvantages mentioned in [35], Sedeno-Noda and Gonzalez-Barrera [36] pointed out a new drawback of the proposed algorithms in the literature [33–35], posed as the need for implicitly or explicitly enlarging the original given network. Then, they proposed a completely new algorithm and showed it to be more efficient than the existing algorithms [33–35] through an extensive experiment involving some real networks such as USA road networks. Shawi et al. [37] considered a transportation network with each arc showing a road with its individual speed and each node exhibiting an endpoint of a road or an intersection between two roads. They considered the cost of moving along an arc as the distance traversed along the arc times the weight of the arc and found the cost of the quickest path in the network. Giani and Guerriero [38] defined a function to obtain a lower bound for the travel time from each node to every other one, and then considered three large metropolitan networks to show the effectiveness of their introduced lower bound.

From the viewpoint of quality of service, evaluating the ability of a communication network to carry out the transmission requests within a given time threshold is an important issue, and so considering an SFN, Lin [25] extended the quickest path problem to the system reliability problem and introduced quickest path reliability problem. He proposed an algorithm to find all system state vectors by which d units of flow can be transmitted through a single quickest path subject to a time limitation T, and then computed system reliability using the inclusion–exclusion principle [14,15]. Yeh and El Khadiri [26] used the universal generating function method [29] to propose a new efficient algorithm for solving the problem. They showed their algorithm to be more efficient than the one proposed by Lin [25]. In addition to time, cost is another important issue in evaluating the system reliability of an SFN. Also, clearly the transmission time will be decreased when the flow is transmitted through more than one minimal path. For this, considering the budget limit, Lin [27] extended the problem to the case of two separate minimal paths (SMPs) and evaluated the probability of transmitting d units of flow through two SMPs from the source node to the sink node satisfying time and budget limits. Nevertheless, there is room for improvement upon the solution methods for the 2 SMPs reliability problem. Here, we present some new results and use them to propose an improved algorithm. We show the algorithm to be more efficient than the recently presented one in [27] on the basis of its complexity results and with respect to the obtained numerical results in the sense of the performance profile introduced by Dolan and More' [39].

The remainder of our work is organized as follows. In Section 2, the required notations, nomenclature and assumptions are given. Then, some new results are presented and an efficient algorithm for the 2 SMPs reliability problem is proposed. This section also illustrates the proposed algorithm through a benchmark ARPANET example. Moreover, the algorithm is shown to be more efficient than the existing ones based on time complexity in Section 2. We explain how the system reliability can be computed employing the proposed algorithm along with the sum of disjoint product technique in Section 3. In Section 4, we make the efficiency comparisons using the performance profile of Dolan and More' [39] on the results obtained over randomly generated test problems. Section 5 provides our concluding remarks.

2. Basic results and the proposed algorithm

Here, we first state the required notations, nomenclature and assumptions, and then present some new useful results. Afterwards, using the new results, an efficient algorithm is proposed for the two separate paths reliability problem. We illustrate our algorithm through a benchmark ARPANET example. We also show our proposed algorithm to be more efficient than the existing ones in terms of their computing complexities.

2.1. Notations, nomenclature and assumptions

Acronyms

SMPs	separate minimal paths.
SFN	stochastic-flow network.

Notations

n, m	number of nodes and arcs, respectively.
G	$G = G(N, A, L, M, C)$ is a stochastic-flow network, where $N = \{1, 2, \ldots, n\}$ is the set of nodes, $A = \{a_i \mid 1 \le i \le m\}$ is the set of arcs, $L = (l_1, l_2, \ldots, l_m)$ is a vector with l_i denoting the lead time of arc a_i, for $1 \le i \le m$, $M = (M_1, M_2, \ldots, M_m)$ is a vector with M_i denoting the max-capacity of arc a_i, for $1 \le i \le m$, $C = (c_1, \ldots, c_m)$ is a vector with c_i denoting the per unit transmission cost of flow on arc a_i, for $1 \le i \le m$.
X	$X = (x_1, x_2, \ldots, x_m)$ is current system state vector, where x_i denotes the current capacity of arc a_i having a random value from $\{0, 1, \ldots, M_i\}$, for $1 \le i \le m$.
b	available budget for the network.
T	time limit.
P_j	jth SMP, for $j = 1, 2$.
LP_j	lead time of P_j, for $j = 1, 2$.
CP_j	transmission cost of P_j per unit of flow, for $j = 1, 2$.
$KP_j(X)$	capacity of P_j, for $j =$ 1, 2, under system state vector X.
d	a non-negative random integer number giving the required flow to be sent in the network through two SMPs.
D	$D = (d_1, d_2)$ is a *policy vector* with d_j denoting the amount of flow to be sent through P_j, for $j = 1, 2$.
FD	$FD = (fd_1, fd_2)$ is the first obtained *policy vector*.
λ	number of all determined *policy vectors*.
$\bar{r}$	number of all determined *policy vectors* satisfying the budget constraint.
$\varphi(P_j, d_j, X)$	time for transmitting d_j units of flow through P_j, for $j = 1, 2$, under system state vector X.
Γ	$\Gamma = (\gamma_1, \gamma_2)$, where γ_j is the smallest possible capacity of P_j for sending d_j units of flow through P_j within time T, for $j = 1, 2$.