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Network reliability analysis based on percolation theory

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ABSTRACT

In this paper, we propose a new way of looking at the reliability of a network using percolation theory. In this new view, a network failure can be regarded as a percolation process and the critical threshold of percolation can be used as network failure criterion linked to the operational settings under control. To demonstrate our approach, we consider both random network models and real networks with different nodes and/or edges lifetime distributions. We study numerically and theoretically the network reliability and find that the network reliability can be solved as a voting system with threshold given by percolation theory. Then we find that the average lifetime of random network increases linearly with the average lifetime of its nodes with uniform life distributions. Furthermore, the average lifetime of the network becomes saturated when system size is increased. Finally, we demonstrate our method on the transmission network system of IEEE 14 bus.

1. Introduction

In modern society, technological networks are pervasive as they provide essential services including materials [1,2], energy [3,4], information [5] and even social communication [6]. It is not surprising, then, that network reliability is receiving particular attention, on one side as a value requested by the users and on the other side as a challenge for the service providers and network operators. One way to address the problem is to consider the structure connectivity of the network as a graph $\Gamma(V, E)$ consisting of a vertex set $V=\{v_1, v_2,\ldots, v_n\}$ and an arc set $E=\{e_1, e_2,\ldots, e_m\}$. Within this abstraction, terminal reliability can be defined as the probability of achieving connectivity from the source nodes to the terminal nodes [7]. The terminal reliability of networks can be characterized by assessment methodologies [8] such as Reliability Block Diagram (RBD), Fault Tree Analysis (FTA) [9] and so on. Typical algorithms for computing terminal reliability include the state enumeration method [10], sum of disjoint products method [11], factorization method [12], minimal cuts method [13] and cellular automata [14,15].

However, in the consideration of terminal connectivity, the identification of the operational limits of a network is missing [8], where a critical fraction of functional components to sustain the network is considered instead of studying paths in the terminal reliability. Percolation theory [16,17] provides us with an opportunity to overcome this gap, by referring network failure to the situation whereby a critical fraction of network components have failed [18–20]. In the percolation theory, the failure of a node/edge of network is modeled by removal. As the removal of nodes/edges increases, the network undergoes a transition from the phase of connectivity (functional network) to the phase of dis-connectivity (nonfunctional network). The probability threshold signifying this phase transition can be found theoretically or computed numerically by percolation theory. The probability threshold can be used as a statistical indicator for the operational limits of the network, which is not considered in traditional terminal reliability analysis. Thus, percolation theory, based on statistical physics, can help to understand the macroscopic failure behavior of networks in relation to the microscopic states of the network components. It can address questions of practical interest such as "how many failed nodes/edges will break down the whole network?"

In this paper, we define "network reliability" by using concepts of percolation theory and exploit the related statistical physics techniques to calculate it. We analyze the network failure process and network reliability properties by percolation theory, providing a new framework for network reliability analysis. In Section 2, we further explain the operational limits of a network. In Section 3, we relate the network reliability problem to percolation theory. In Section 4, we analyze theoretically the network reliability and lifetime distribution, referring to random networks. In Section 5,

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Notation

V	a set of vertices
E	a set of arcs
$\Gamma\ (V, E)$	a network defined as an undirected graph with V, E
N	the total number of nodes in a network
C_N^i	the binomial coefficient
$<a>$	the average value of the random variable a
p	the probability that a node/edge is functional
p_c	the percolation threshold
T_s	the average lifetime of the network
$a*b$	product of a and b
$[a]$	the largest integer less than or equal to a

we present simulation results, which are extended to real networks in Section 6.

To accompany the reader throughout the study of the paper, we anticipate here a number of definitions:

Definition 1. Random Network (Erdös-Rényi (ER) Network [21]): A graph with N vertices can have C_N^2 pairs at most. To generate a random network, we first build N nodes. Then we connect each pair of nodes with the same probability, p. In this way, a random network (N, p) can be constructed finally and the networks will become more connected with increasing p. Fig. 1 gives an example of random network with $N=150$, $p=1/75$.

Definition 2. Degree of node i, k_i: the number of links that belong to node i. $<k>$: the average value of the degree, which is the sum of node's degree divided by the number of nodes in the network. In Fig. 1, the average degree of the network $<k>=2$.

Definition 3. Cluster: a connected set of nodes, within which there is a path between any pair of nodes. G represents the size of the largest (giant) cluster in the network, while SG represents the size of the second largest cluster. In Fig. 1, G is the cluster consisting of the red nodes, and SG is the cluster consisting of the blue nodes.

2. The operational limits of a network

Given a network such as communication networks or power grid, many studies focused on the terminal reliability between pair of nodes in the network. The terminal reliability includes the two-terminal reliability, K-terminal reliability and all-terminal reliability. These studies investigate the connectivity between the origin and destination of a given pair from the viewpoint of network users.

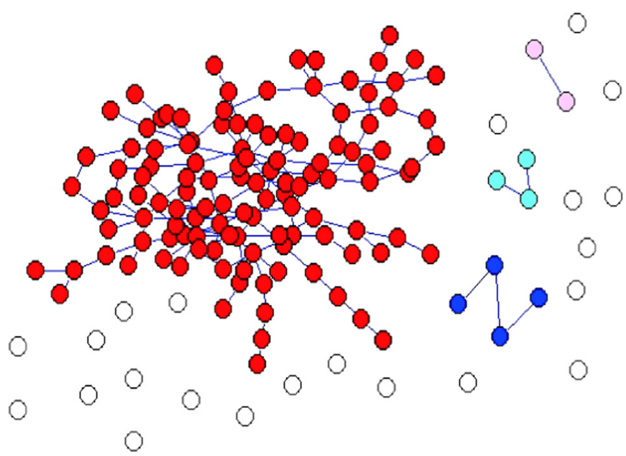

Fig. 1. Random Network with $N=150$, $<k>=2$. G is the cluster consisting of the red nodes, and SG is the cluster consisting of the blue nodes. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

However, system operators cannot put all of the weight onto the service quality of a single user or a portion of users, and rather care about questions of practical interest such as "how many failed nodes/edges will break down the whole network?" Accordingly, we investigate the macroscopic status of network reliability by defining "the operational limits of a network" in this manuscript. When the classical methods of terminal reliability are implemented to answer above questions, the "combinatorial explosion" problem [22,23] usually occurs when the number of possible system states increases exponentially or even faster with the system size. In an attempt to overcome the "combinatorial explosion" problem in association with the assessment of network reliability, we consider the phase transition threshold of the network failure process as an indicator of network connectivity loss, and use percolation theory to identify the critical point and calculate reliability indexes correspondingly defined. Instead of focusing on verifying the existence of paths connecting source and target nodes, we study the network reliability in a system view.

Percolation theory has been widely applied in the field of complex networks [18–20]. Based on this, many studies have allowed revealing important network characteristics, including vulnerability analysis of different types of complex networks. Percolation actually describes a phase transition process of network failure, whose critical point distinguishes the network from connected to disconnected. Percolation theory makes use of statistical physics principles and graph theory to analyze such change in the structure of a complex network. Specific examples of problems, which can be described and analyzed by percolation theory, are the robustness of networks against random failures and targeted attacks [24,25], the interdependent systems [26], the spreading of infectious diseases [27].

3. Network reliability analysis based on percolation theory

In the following, by taking into account the lifetime of the network nodes, we study how the global network connectivity changes during a process of nodes and/or edges failure and measure the network reliability $R_s(t)$ and lifetime distribution $f_s(t)$ as defined with respect to the critical point of the network percolation process. Let $R(t)$ be the probability that a node/edge is functional at a given time t, i.e. the node/edge reliability at time t. A fraction $1-p=1-R(t)$ of nodes/edges will fail according to their reliability and as the failure process proceeds, clusters of connected nodes form as they are cut off from the main (giant) network cluster (whose size is indicated as G). Then, as further nodes/edges fail, the network gradually fragments into many finite clusters. If $R(t)$ is below a critical value p_c, the main network cluster does not exist anymore and only small isolated clusters exit: we define the instant at which this occurs as the lifetime of the network. With the network topology information, this critical value p_c can be calculated according to percolation theory [18–20], which distinguishes the network from being connected to disconnected.