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Enhancing product robustness in reliability-based design optimization

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ABSTRACT

Different types of uncertainties need to be addressed in a product design optimization process. In this paper, the uncertainties in both product design variables and environmental noise variables are considered. The reliability-based design optimization (RBDO) is integrated with robust product design (RPD) to concurrently reduce the production cost and the long-term operation cost, including quality loss, in the process of product design. This problem leads to a multi-objective optimization with probabilistic constraints. In addition, the model uncertainties associated with a surrogate model that is derived from numerical computation methods, such as finite element analysis, is addressed. A hierarchical experimental design approach, augmented by a sequential sampling strategy, is proposed to construct the response surface of product performance function for finding optimal design solutions. The proposed method is demonstrated through an engineering example.

1. Introduction

Uncertainties that exist in a product's design-manufacturing-usage life-cycle need to be addressed at the product's early design stage. The traditional reliability-based design optimization (RBDO) method deals with the uncertainties in product design variables by treating them as random variables and by formulating a probabilistic constraint on product performance function so as to satisfy its reliability (or safety) requirement. Although accounting for environmental noise and its effects on product quality has been a part of the RBDO methodology, this issue was often ignored in previous case studies. On the other hand, the theory of robust product design (RPD) explores the interaction between noise variables and design variables for the reduction of the total variance of product performance. In this paper, these two perspectives of product design optimization are integrated into a unifying design framework and, subsequently, an efficient computational strategy for generating good design candidates is proposed.

Our research contributions consist of the following components: first, we define the robustness of a product from the quality engineer's perspective; that is, it is the variance of product performance due to environmental noise. A multi-objective optimization framework is proposed for combining the considerations of product quality and product reliability (or safety) in the product design optimization process. Second, as product performance is often evaluated by a computer model in modern design, we assume that the product performance function is implicit and propose a hierarchical experimentation method for constructing the metamodel of product performance function at different noise levels. This is a response surface approach to robust design; however, it is applied on computer experiments, instead of physical experiments. Third, a sequential sampling technique is developed for searching the robustness-oriented most probable design points. Our proposed algorithm is able to explicitly incorporate the modeling error of metamodel and the random effect from noise variables into the product design optimization process using a small number of experimental runs.

In the remainder of this section, the basic concepts of RBDO and RPD are introduced and the different perspectives of these two design philosophies are discussed. Through an example, we demonstrate the benefits of combining these two philosophies for product design. In Section 2 a new unifying framework of design optimization with the considerations of both product reliability and product robustness is proposed. We investigate the problem of using a surrogate model, or metamodel, to approximate product performance function that can only be evaluated by computer experiments. This problem is often met in complex engineering designs, especially at a product's early design stage. To solve the optimization problem, we derive a hierarchical experimentation and sequential sampling strategy for updating the metamodel, which are presented in Section 3. The solution of the illustrative

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example is thoroughly discussed in Section 4. Finally, the paper is concluded in Section 5.

1.1. Reliability-based design optimization

RBDO concerns with the effects of random design variables and noise variables on product performance and product's production cost. It requires the product performance function to satisfy a performance criterion that is derived from the product's reliability or safety concern. As both design variables and noise variables are random, this performance requirement is formulated as a probabilistic constraint, which is referred as the reliability constraint. The objective function in RBDO is the mean production cost, which includes, e.g., material cost, manufacturing cost, labor cost, etc. A generic model of RBDO is as follows:

$$\text{Minimize}_{\mathbf{d},\boldsymbol{\mu}_x} \quad E[f(\mathbf{d},\mathbf{x},\mathbf{p})]$$

$$\text{Subject to} \quad Pr[G_i(\mathbf{d},\mathbf{x},\mathbf{p}) \geq 0] \geq R_i, \quad i = 1,2,\ldots,m$$

$$\mathbf{d}^{\mathbf{L}} \leq \mathbf{d} \leq \mathbf{d}^{\mathbf{U}}, \boldsymbol{\mu}_{\mathbf{x}}^{\mathbf{L}} \leq \boldsymbol{\mu}_{\mathbf{x}} \leq \boldsymbol{\mu}_{\mathbf{x}}^{\mathbf{U}} \tag{1}$$

In (1) $\mathbf{d}$ denotes a vector of deterministic design variables, $\mathbf{x}$ denotes a vector of stochastic design variables, and $\mathbf{p}$ denotes a vector of noise variables. A stochastic design variable is controllable in the sense that its mean value can be specified by the designer even if its variance may not be reduced, due to, for example, natural variations in materials and in manufacturing. A noise variable is only observable, but not controllable. The function, $f(\mathbf{d},\mathbf{x},\mathbf{p})$, is a production cost function. Due to the random nature of design variables and noise variables, the objective function of RBDO is the mean cost function $E[f(\mathbf{d},\mathbf{x},\mathbf{p})]$, which is usually replaced by its first-order Taylor expansion approximation, $f(\mathbf{d},\boldsymbol{\mu}_{\mathbf{x}},\boldsymbol{\mu}_{\mathbf{p}})$. The function $G_i(\mathbf{d},\mathbf{x},\mathbf{p})$ in the probabilistic constraint in Model (1) is one of the product's performance functions. To define a reliable product, it requires $G_i \geq 0$. Thus, $G_i < 0$ represents the failure of product's ith function and $G_i = 0$ defines a limit-state surface, which is the boundary between success and failure. The inequality constraint $Pr[G_i \geq 0] \geq R_i$ defines the product's reliability level to be larger or equal to the targeted reliability level, R_i.

From a designer's perspective, the main task of RBDO is to keep the designed product safe or reliable with minimum production cost. Therefore, the right hand side of the constraint function in Model (1) specifies the required probability of an acceptable product performance. The computational work involved in solving an RBDO problem is dominated by the evaluation of this probabilistic constraint function. The methods developed for solving an RBDO problem include double-loop methods [1], decoupled-loop methods [2,3] and single-loop methods [4–6]. Oftentimes in practice the effects of noise variables in RBDO on the robustness of performance functions are ignored or are treated as pure additive variability, thus having no impact on the problem solution. However, two issues arise when ignoring noise variables [7]: the design feasibility could be in doubt because the effect of extra variation from noise variables may lead to the shrinkage of feasible region, and more importantly, the transmitted variation from noise variables to the performance function can result in the deterioration of product quality. From a quality engineer's perspective, reducing variance is the ultimate goal of product quality control, as a large variance will lead to large potential cost, such as warranty/repair cost, which is the long-term cost of quality-related deficiencies. In order to reduce the impact of noise variables on both product quality and design feasibility, a robust design process needs to be implemented.

1.2. Robust product design

Robustness is the state where a product or process' performance is less sensitive to the factors that may cause variation [8]. Robust product design is an approach for improving the quality of a product by minimizing the effect of the cause of variation without eliminating the source of variation [9]. Noise variables formulated in RBDO are uncontrollable, but their effects on product performance can be mitigated by adjusting controllable design variables. The solution is made possible by exploiting the interaction between design variables and noise variables. In Model (1), however, the robustness of product performance is not directly investigated. It is commonly seen in the literature that the noise variable is either directly replaced by its mean value (0) or it is simply treated as same as another random decision variable. Some previous studies defined the product robustness as the variability of the production cost function due to these random variables (see, e.g., [4,10,11]). This is different from our study where the robustness is defined as the sensitivity of product performance function to noise variables.

To understand the effects of model uncertainty in robust design, Apley [12] assigned normal distributions to noise variables and proposed a Bayesian framework for quantifying the impact of interpolation uncertainty on the robust design objective. Rangavajhala et al. [13] examined the challenge of equality constraints in robust design optimization. Zaman et al. [14] analyzed the impact of non-design epistemic variables on robustness-based design optimization. Tang [15] developed a feasible robustness index and integrated it into the RBDO formulation. Some authors considered the worst case analysis and the moment matching method for robust design [7–9]. For example, Xu [16] employed the worst case analysis of maximum design parameter deviation and proposed a robust design model based on maximum variation estimation.

Taguchi's method is one of the most popular methods employed in robust design. Based on different phrases of design process, Taguchi provided a three-stage process [17]: system design, parameter design, and tolerance design, while parameter design is arguably the most important one. During a parameter design, product design parameters are optimized for improving product quality. Unlike the ordinary design optimization, Taguchi's parameter robust design accounts for the product performance variation due to noise factors. Suppose $G(\mathbf{x},\mathbf{p})$ is a performance function, where $\mathbf{x}$ and $\mathbf{p}$ are controllable variables and noise variables, respectively. A signal-to-noise ratio (SNR), as a measure of quality loss, is defined as follows:

$$\text{SNR} = -10 \log[\text{MSD}] \tag{2}$$

where $\text{MSD} = (1/k)\sum_{i=1}^{k}[G(\mathbf{x}_i,\mathbf{p}_i) - G_T]^2$ is the sum of mean square deviations of the performance function from its targeted value. The function $G(\mathbf{x}_i,\mathbf{p}_i)$ denotes the product performance of a single sample. In order to characterize this function, the techniques of design of experiments (DOE) are employed. By evaluating different designs based on experimental results, the best combination of control factors is found. However, the orthogonal array designs proposed by Taguchi are defined in discrete space and they cannot be easily extended to a wider design range. In addition, it is not an efficient method for a problem with a large set of experimental factors [18].

An alternative approach is called robust design optimization (RDO), which directly minimizes the variance of product performance function by exploiting interaction effects of design variables and noise variables. A generic form of RDO model is given by

$$\text{Minimize} \quad \text{Var}[G(\mathbf{d},\mathbf{x},\mathbf{p})]$$

$$\text{Subject to} \quad E[G(\mathbf{d},\mathbf{x},\mathbf{p})] \geq G_T$$