Reliability Engineering and System Safety 134 (2015) 198–207

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Optimal maintenance policy and residual life estimation for a slowly degrading system subject to condition monitoring

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ARTICLE INFO

Article history:
Received 7 August 2013
Received in revised form
9 October 2014
Accepted 13 October 2014
Available online 28 October 2014

Keywords:
Condition monitoring
Preventive maintenance
Semi-Markov decision process
Autoregressive model with time effect
Control-limit policy
Residual life estimation

ABSTRACT

In this paper, we present an optimal preventive maintenance policy and develop a procedure for residual life estimation for a slowly degrading system subject to soft failure and condition monitoring at equidistant, discrete time epochs. An autoregressive model with time effect is considered to describe the system degradation, which utilizes both the system current age and the previous state observations. The class of control-limit maintenance policies with two different inspection strategies is considered, and the optimization problem is formulated and solved in a semi-Markov decision process framework. The objective is to minimize the long-run expected average cost. A formula for the mean residual life is derived for the proposed degradation model and a control limit policy, which enables the estimation of the remaining useful life and early maintenance planning based on the observed degradation process. An example is presented to demonstrate the effectiveness of the proposed method.

1. Introduction

The performance of engineering systems is usually affected by a degradation process that occurs gradually over time. When the system state defined by the degradation level reaches a predetermined threshold, the system is no longer assumed to be able to function satisfactorily or safely and it should be stopped and replaced, although no physical failure is observed. This so-called soft failure resulted from degradation might incur high costs (e.g. due to production losses, quality decrease) and/or safety hazards (e.g. the cracked structure can fail when the safety standard is violated). For this reason, a preventive maintenance should be performed before soft failure occurs which may be required by a law and it typically has a higher economic and safety significance than a corrective maintenance which only takes place when the failure is observed. Many real systems are maintained periodically applying an age-based or length-of-usage-based policy which can be highly ineffective or hazardous, e.g. when applied to power plants and civil structures such as bridges, tunnels, buildings, etc. [17].

If the degradation state can be directly measured by discrete or continuous condition monitoring, it is desirable to make the maintenance decision based on the actual degradation state of the system (see e.g. [1]). In this paper, we consider a system subject to degradation and soft failure. The degradation process is monitored through perfect inspections which fully reveal the system condition. If the system's degradation level identified by inspection exceeds its failure threshold, a corrective replacement is performed. The system will be preventively replaced if its observed degradation state exceeds a pre-determined control limit and it is left operational until next inspection if its degradation level is below the control limit.

The essential part in analysis and computation of the optimal maintenance policies for a degrading system is the degradation modeling. The degradation process could be modeled by a discrete-state or a continuous-state stochastic process, or by their combinations. A three-state stochastic degradation model was considered, e.g. by Makis [8], Kim et al. [6] and Wang [19]. Wang [19] assumed arbitrary random sojourn times in each state in order to cover a more general situation. It is true that degradation analysis with Markovian assumption may suffer from some limitations (Si et al. [15]), however, convenience from the modeling estimation and mathematical perspectives makes Markov and semi-Markov processes still popular and prominent among degradation models. The discrete states in Markovian-based models could be representative of system degradation stages (Makis [8], Kim et al. [6], Makis and Jiang [10], Zhou et al. [23], and Giorgio et al. [4]), or represent the stochastic covariate values that directly affect the degradation trend (Makis and Jardine [9], Zhao et al. [22], Kharoufeh and Cox [5], Xiang et al. [20]). Makis and Jiang [10] considered a discrete-continuous maintenance model based on a continuous-time hidden Markov state process and a discrete

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http://dx.doi.org/10.1016/j.ress.2014.10.015
0951-8320/

observation process to deal with the real situation when the condition-monitoring data is collected in discrete times but the degradation process is continuous over time. In Zhou et al. [23], continuous-state degradation process was first assumed. By Monte Carlo-based density projection method, the infinite continuous state space was mapped into a finite dimensional process of "beliefs", which were then used as discrete states in a Markov decision process to determine the optimal maintenance policy. A similar idea of using the Markovian degradation model to approximate the continuous-state degradation process can be found in Giorgio et al. [4]. Markovian-based models have also been used to model the evolution of covariates in degradation processes (applications can be found in Makis and Jardine [9], Makis et al. [11], Zhao et al. [22], Kharoufeh and Cox [5] and Xiang et al. [20]).

In contrast with discrete-state stochastic models, continuous-state stochastic models assume that the evolution of degradation states is continuous over time. In previous maintenance modeling of degrading systems, path-dependent degradation models have been commonly used, among which the age-dependent linear path and the exponential path have been the most popular assumptions for degradation trend (Wang [18], You et al. [21]). Wang [18] derived an optimal control limit and monitoring interval based on a linear degradation model with a Weibull-based random-effect slope and an i.i.d. error term. You et al. [21] used exponential degradation path for replacement decision making, with both offline and real-time average system availability optimization in one replacement cycle as an objective of maintenance control.

A combination of a Markov process and path-dependent stochastic degradation models has also been considered recently (see e.g. [3] and [2]). This modeling approach is also the inspiration for our work. Elwany et al. [3] introduced an idea of modeling age-dependent degradation model as a non-stationary Markov process considering a structured replacement policy. The underlying age-dependent degradation model is re-estimated at each inspection by condition-monitoring information, and the maintenance policy is optimized using a Markov decision process. Curcurù et al. [2] described a degradation process by a first order autoregressive model with drift, which then can be treated as a Markovian degradation process.

In this paper, we will focus on developing a control-limit optimal maintenance policy for a slowly degrading system based on an autoregressive model with time effect. The degradation trend in our proposed model is assumed to be dependent on the previous state observation which is different from previously considered age-dependent degradation models. This assumption is more realistic and appropriate in many situations, for example, the crack propagation rate will be higher when the current crack length is larger. We add a time effect to an autoregressive model to describe the influence of age on the degradation process. The control limit as a preventive replacement threshold, the first inspection time and the length of the subsequent regular inspecting intervals are decision variables in our maintenance policy, which will be determined by formulating and analyzing the decision problem in a semi-Markov decision process framework. The effectiveness of the proposed method will be demonstrated using a real laser degradation data set coming from Meeker and Escobar [12]. We note that the autoregressive model with the time effect has not been considered before to model system degradation. Also, the calculation of the optimal control policies under different inspection strategies and the derivation of the formulas for the residual life estimation for this model are new and have not appeared in the literature.

The rest of the paper is organized as follows. Section 2 introduces the degrading systems described by autoregressive models with time effect. Section 3 develops the optimal maintenance policy based on the proposed degradation model. Comparison between the proposed maintenance policy and conventional age-based maintenance policy is in Section 4. In Section 5, we develop the formulas for the estimation of the residual life of the slowly degrading system using our proposed degradation model and the control-limit policy illustrated by an example. Conclusions and suggestions for future research are in Section 6.

2. Degrading systems described by autoregressive models with time effect

We assume that the degradation state of the system can only be known at discrete inspection times, which is the case in many real applications. The degradation process starts from a known initial state $Y_0 = y_0$, and it is monitored through regular periodic inspections with inspection interval h. Let Y_n denote the degradation state observed at inspection time $t_n = nh$ $(n = 1, 2, 3, \ldots)$. We consider the following autoregressive model with time effect to describe the degradation process:

$$Y_n - \delta_0 = \beta t_n + \sum_{r=1}^{p} \varphi_r (Y_{n-r} - \delta_0) + \varepsilon_n, p = 0, 1, 2, \ldots, \quad n = 1, 2, 3, \ldots \tag{1}$$

where $\{\varepsilon_n\}$ are i.i.d. $N(0, \sigma^2)$. The model coefficients $(p, \varphi_r, \delta_0, \beta, \sigma^2)$ are unknown and need to be estimated. We set $\delta = \delta_0 - \Sigma_{r=1}^{p} \varphi_r \delta_0$ and then write Eq. (1) in standard form:

$$Y_n = \delta + \beta t_n + \sum_{r=1}^{p} \varphi_r Y_{n-r} + \varepsilon_n \ , \quad p = 0, 1, 2, \ldots, \quad n = 1, 2, 3, \ldots \tag{2}$$

The estimation of the model coefficients starts with determining the model order p. Assume that we have M histories of the system. For the l th data history, we denote the number of inspections by m_l, the observed system states by $\{y_0, y_1^l, y_2^l, \ldots, y_{m_l}^l\}$ where y_0 is the same for all histories, and the inspection times by $\{t_1^l, t_2^l, \ldots, t_{m_l}^l\}$, $l = 1, 2, \ldots, M$. So that for the M observed data histories, we have the regression representation $\mathbf{W} = \mathbf{VA} + \mathbf{E}$, where

$$\mathbf{W}' = [y_{m_M}^M \ldots y_p^M \ldots y_{m_1}^1 \ldots y_p^1]$$
$$\mathbf{E}' = [\varepsilon_{m_N}^N \ldots \varepsilon_p^N \ldots \varepsilon_{m_1}^1 \ldots \varepsilon_p^1]$$
$$\mathbf{A}' = \begin{bmatrix} \delta & \beta & \varphi_1 & \cdots & \varphi_p \end{bmatrix}$$
$$\mathbf{V}' = \begin{bmatrix} 1 & & 1 & & 1 & & 1 \\ t_{m_N}^M & & t_p^M & & t_{m_M}^1 & & t_p^1 \\ y_{m_M-1}^M & & y_{p-1}^{N_1+M_1} & & y_{m_M-1}^1 & & y_{p-1}^1 \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ y_{m_M-p}^M & & y_0 & & y_{m_M-p}^1 & & y_0 \end{bmatrix}. \tag{3}$$

The least squares estimate of $\mathbf{A}$ is given by (see e.g. Reinsel [13])

$$\hat{\mathbf{A}} = (\mathbf{V}'\mathbf{V})^{-1}\mathbf{V}'\mathbf{W}. \tag{4}$$

where $T = \Sigma_{l=1}^{M}(m_l - p)$ is the total number of available observations of the degradation state.

The estimate of the model order $p \in N$ is obtained by testing $H_0 : \varphi_p = 0$ against $H_a : \varphi_p \neq 0$ and using the likelihood ratio statistic given by

$$D_p = -(T - p - 1 - 1/2) \ln\left(\frac{\det(S_p)}{\det(S_{p-1})}\right). \tag{5}$$

where $S_p = (\mathbf{W} - V\hat{\mathbf{A}})'(\mathbf{W} - V\hat{\mathbf{A}})$ is the residual sum of squares matrix obtained from fitting the autoregressive model with time effect of order $p \in N$. For large $T = \Sigma_{l=1}^{M}(m_l - p)$, if $\varphi_p = 0$, D_p converges in distribution to the chi-square distribution with 1 degree of freedom (see e.g. Reinsel [13]). Thus, for significance level $\alpha \in (0, 1)$, we reject $\varphi_p = 0$ if $D_p > \chi_{1,\alpha}^2$.