Reliability Engineering and System Safety 126 (2014) 44–53

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

A response surface method for stochastic dynamic analysis

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ARTICLE INFO

Article history:
Received 4 June 2012
Received in revised form
20 September 2013
Accepted 5 January 2014
Available online 17 January 2014

Keywords:
Stochastic dynamics
FORM
Response surface
Linearization
Bouc–Wen

ABSTRACT

A Response Surface (RS) strategy is presented for the evaluation of the response statistics of dynamic systems subjected to stochastic excitation. The proposed approach adopts a strategy based on the High Dimensional Model Representation (HDMR), which gives a Gaussian Model (GM) of the response. The GM requires only a reduced number of analyses which can be adopted for all the degrees of freedom of a MDOF dynamic system and it can be successfully adopted for weakly nonlinear dynamic systems.

For more strongly nonlinear systems a Non-Gaussian approximation may be necessary for the highest response thresholds. In this paper this issue is accomplished through the FORM solution, and the design point is obtained by using a response surface method recently proposed by the author and Der Kiureghian to this aim. The latter response surface is based on a variant of the Model Correction Factor Method (MCFM), which is here applied by using as a starting model the GM itself.

In many applications of engineering interest, both the input and the response processes are stationary, so that the stochastic excitation through the Fourier series can be modeled in terms of the underlying Power Spectral Density (PSD). In these cases, it is seen that the dynamic computations required by the proposed approach can decrease significantly. The application to SDOF and MDOF hysteretic systems shows the effectiveness of the presented method.

1. Introduction

The aim of the stochastic dynamic analysis is the evaluation of the response of dynamic systems subjected to stochastic input. If the stochastic excitation follows a Gaussian distribution and the dynamic system is linear, then the response follows a Gaussian distribution, too, and it is relatively easy to be determined. Conversely, the response of a nonlinear dynamic system follows a Non-Gaussian distribution, and its determination is a very complicated task to be accomplished.

In the last decades a lot of research has been devoted to this topic, however most approaches are not easily applicable to general non-linear MDOF systems, and so they are difficult to apply in practice.

These drawbacks are not shared by the Equivalent Linearization Method (ELM) [1], which has gained wide popularity because of its versatility and applicability, see among others, [2,3]. The basic idea is to replace the original non-linear system by an equivalent linear one, whose determination is performed by minimizing the difference between the two systems in some statistical sense. The ELM exhibits different forms based on the adopted probability density function for the evaluation of the coefficients that appear in the linearized system. Usually the Gaussian distribution is adopted, which allows to approximate the second order moments of the response. Against relatively little numerical efforts, unfortunately it gives accurate results for weakly non-linear systems only [1,4]. Moreover, the ELM generally cannot approximate adequately the distribution probability of the response, especially in the tail region. Therefore some response statistics as crossing rates and first passage probability will be inaccurate at high thresholds for systems strongly nonlinear.

The most robust procedure is given by the Monte Carlo Simulation (MCS), which is however strongly demanding in its crude form. For this reason, recently some smart simulation techniques have been proposed, among the other we recall the subset simulation [5], line sampling [6], asymptotic sampling [7].

Promising results are given from the application to the non-linear stochastic dynamic analysis of the analytical methods of structural reliability, particularly the first-order reliability method (FORM) [8,9]. At first, the stochastic input is discretized into a large number of standard normal random variables. The tail probability is defined as the probability that the response of the dynamic system is greater than a chosen threshold x at fixed time instant t. In this way, for given x and t the dynamic problem may be solved by using FORM. The knowledge of the design point is of great importance, because: (i) it corresponds to the most likely realization of the stochastic input that gives rise to the tail exceedance event, and therefore it defines a critical excitation for the system,

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http://dx.doi.org/10.1016/j.ress.2014.01.003

(ii) it gives the FORM solution, which has been shown to give very good approximations of the tail probability in many cases of practical interest [10–12], (iii) it allows the application of the recently developed tail equivalent linearization method (TELM) [13–17].

However, in high-dimensional spaces, as the one here analyzed, the evaluation of the design point is a challenging task. Indeed, the design point is obtained as a solution of a constrained optimization problem [18,19]. For some material models, the problem is not numerically very smooth, and the finite element code fails to produce a result due to lack of numerical convergence, so that some suitable remedying strategies have to be adopted [20]. Moreover, for the solution of the optimization problem gradient-based procedures are usually adopted, which require repeated evaluation of the response gradient. Typically, the finite elements codes do not provide these gradients, which can be approximated by the finite differences method (FDM). By using FDM, each gradient at each iteration of the iterative procedure requires $n+1$ nonlinear dynamic computations, n being the number of random variables, and consequently the cost of computation may be excessive. Moreover the selection of the perturbation parameter in FDM is questionable, and in particular in high-dimensional spaces the accuracy of the response gradient may be lost. Consistency, accuracy and efficiency of the gradients can be achieved by using the direct differentiation method (DDM) [21–23], which involves analytical differentiation of the discretized response equations. However, the DDM requires alterations at the finite element (FE) code level, and it is necessary to use a DDM-enabled software, such as OpenSees [24].

To reduce the computational cost and using an existing FE code as a “black-box”, an alternative strategy is given from the Response Surface Methodology (RSM), which builds a surrogate model of the target Limit State Function (LSF), defined in a simple and explicit mathematical form [25–28]. Once the Response Surface (RS) is built, it is possible to substitute the RS for the target LSF, and then it is no longer necessary to run demanding finite element analyses.

In this paper we used a novel response surface strategy based on the High Dimensional Model Representation (HDMR) [29]. The HDMR is a set of analysis tools for capturing the high-dimensional relationships between sets of input and output model variables. The existing HDMR-based response surfaces [30] require for the first-order representation of the limit state function on average 5–7 points for each random variable, so for the problem at hand a huge computational effort arises, since we have a different limit state function for each response threshold. In this paper we present a suitable chosen response surface requiring only 2 points for each random variable, reducing significantly the dynamic computations.

The proposed application of the HDMR gives a Gaussian Model (GM) for the system response and it gives quite good approximations for weakly nonlinear dynamic systems. For systems with strong nonlinearities it is likely that especially for the highest thresholds the FORM solution outperforms the GM. In any case, the design point of the GM is likely to be close to the design point of the original problem, so it can represent a good starting point for FORM. It is expected that in general the algorithm converges with only a few iterations. In this paper, to reduce further the computational cost, we adopted the “improved Model Correction Factor Method” (iMCFM), developed by the author with A. Der Kiureghian in recent papers [11,12].

The paper is organized as follows: in Sections 2 and 3 we present the nonlinear stochastic dynamic problem by using the proposed response surface strategy giving rise to the GM, in Section 4 its improvement through the MCFM is discussed, and finally in Section 5 the method is applied to hysteretic systems, showing its accuracy and efficiency.

2. Discretization of the stochastic input

The proposed approach requires the preliminary discretization of the stochastic input into a set of standard normal random variables. Several formulations for this purpose are available, see Der Kiureghian [9] for a brief review. For a zero-mean, Gaussian excitation process, all representations have the form

$$F(t,\mathbf{u}) = \sum_{i=1}^{n^{(\tau)}} s_i^{(\tau)}(t)u_i = \mathbf{s}^{(\tau)}(t) \times \mathbf{u} \quad (1)$$

where $n=n^{(\tau)}$ is a measure of the resolution of the discretization, $\mathbf{u}=\{u_1,u_2,\dots,u_n\}^T$ is an n-vector of standard normal random variables, $\mathbf{s}^{(\tau)}(t)=\{s_1^{(\tau)}(t) \quad s_2^{(\tau)}(t) \quad \dots \quad s_n^{(\tau)}(t)\}^T$ is an n-vector of deterministic shape functions dependent on the covariance function of the process. In (1) the superscript “τ” refers to the discretization of the input in the time domain [8–14,16,17]. In the special case of $s_i^{(\tau)}(t)=\sigma \times \Delta t \times \delta(t-t_i)$, where $\delta(t)$ is the Dirac delta function and $t_i = i\Delta t$, $i=1,2,\dots,n$, are a set of equally spaced time points, then, $F(t,\mathbf{u})$ represents a discretized band-limited white noise excitation of intensity S_W, of variance $\sigma_F^2=(2\pi S_W)/\Delta t$ and with cut-off frequency $\omega_c=\pi/\Delta t$.

If the excitation is a stationary process defined by the power spectral density (PSD) function $S_F(\omega)$, through the discrete Fourier series the stochastic input can be expressed as [15–17]

$$F(t,\mathbf{u}) = \sum_{k=1}^{n^{(\omega)}} F_k(t,\mathbf{u}) = \sum_{k=1}^{n^{(\omega)}} \sqrt{2S_F(\omega_k)\Delta\omega}[\cos(\omega_k t)u'_k + \sin(\omega_k t)u''_k]$$
$$= \sum_{k=1}^{n^{(\omega)}} s_k^{\prime(\omega)}(t)u'_k + s_k^{\prime\prime(\omega)}(t)u''_k = \mathbf{s}^{(\omega)}(t) \times \mathbf{u} \quad (2)$$

where $n=2n^{(\omega)}$ is a measure of the resolution of the discretization, $\mathbf{u}=\{u'_1,u''_1,u'_2,u''_2,\dots,u'_{n/2},u''_{n/2}\}^T$ is an n-vector of standard normal random variables, $\mathbf{s}^{(\omega)}(t)=\{s'_1(\omega) \quad s''^{(\omega)}_1 \quad \dots \quad s'_{n/2}(\omega) \quad s''^{(\omega)}_{n/2}\}^T$ is an n-vector of deterministic shape functions dependent on the PSD of the process, being $s'_k(\omega)=\sqrt{2S_F(\omega_k)\Delta\omega}\cos(\omega_k t)$ and $s''^{(\omega)}_k=\sqrt{2S_F(\omega_k)\Delta\omega}\sin(\omega_k t)$; here the superscript “$\omega$” refers to the discretization of the input in the frequency domain. To obtain a good approximation of $F(t,\mathbf{u})$ the frequency step is $\Delta\omega \le (2\pi)/t$, being t the time instant when the input process reaches stationarity; if t is equal to the last time instant of (1) and choosing $\Delta\omega=(2\pi)/t$ so that $n^{(\omega)}=n^{(\tau)}/2$, then the (2) corresponds to (1), in the sense that the PSD $S_F(\omega)$ is sampled at the frequencies $0,\omega_1,\omega_2,\dots,\omega_{n/2}$ that we would obtain by applying the discrete Fourier transform (DFT) to each sample of (1). Under these conditions the variance of a band-limited white noise excitation of intensity S_W by using (2) is $\sigma_F^2=(2\pi S_W)/\Delta t$.

3. An alternative Gaussian approximation of the system response

Consider the response of a dynamical system to the excitation in (1)or (2). Owing to the random variables $\mathbf{u}$, the response is stochastic and we denote it as $X(t,\mathbf{u})$. For a specified threshold x and time t, we define the tail probability as $\text{Prob}[X(t,\mathbf{u}) \ge x]$. To apply the tools of the structural reliability theory, we define the limit state function (LSF)

$$g(x,t,\mathbf{u}) = x - X(t,\mathbf{u}) \quad (3)$$