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Importance analysis for reconfigurable systems

Shubin Si [a], Gregory Levitin [b,*], Hongyan Dui [a], Shudong Sun [a]

[a] School of Mechanical Engineering, Northwestern Polytechnical University, P.O. Box 554, Xi'an, Shaanxi 710072, PR China
[b] The Israel Electric Corporation, P.O. Box 10, Haifa 31000, Israel

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ABSTRACT

Importance measures are used in reliability engineering to rank the system components according to their contributions to proper functioning of the entire system and to find the most effective ways of reliability enhancement. Traditionally, the importance measures do not consider the possible change of system structure with the improvement of specific component reliability. However, if a component's reliability changes, the optimal system structure/configuration may also change and the importance of the corresponding component will depend on the chosen structure. When the most promising component reliability improvement is determined, the component importance should be taken into account with respect to the possible structure changes. This paper studies the component reliability importance indices with respect to the changes of the optimal component sequencing. This importance measure indicates the critical components in providing the system reliability enhancement by both enhancing the component's reliability and reconfiguring the system. Examples of linear consecutive-k-out-of-n: F and G systems are considered to demonstrate the change of the component Birnbaum importance with the optimal system reconfiguration. The results show that the change of the importance index corresponds to the change of the system optimal configuration and the importance index can change not monotonically with the variation of the component reliability.

1. Introduction

In several classes of heterogeneous systems the system reliability can be considerably improved by optimal arrangement of its components [1–7]. In such systems two methods of system reliability improvement exist: (1) enhancement of reliability of system components and (2) rearrangement of existing components (for example, relocation/reordering of the components along a line or changing the components' weights) without changing their reliability. These two methods interact with each other because the optimal system configuration (arrangement of the components) depends on the components' reliabilities. When the system can be easily rearranged, reliability enhancement of a specific component can be followed by a rearrangement to achieve the greatest improvement of the entire system reliability. Thus, while studying the possible effect of component reliability enhancement it is important to take into account the optimal system reconfiguration after such improvement.

Optimal configuration problems have been much discussed by Malon [1,2] and Tong [3]. Zuo and Kuo [4] summarized the results available for the invariant optimal design of consecutive k-out-of-n: F systems and identified invariant optimal designs for such systems. Levitin and Lisnianski [5] studied the reliability optimization for weighted voting system by changing the weights of the voting units. Levitin [6–8] studied the optimal sequencing of components in linear consecutively-connected systems, multi-state sliding window systems and heterogeneous standby systems. Jalali et al. [9] obtained the proof of the invariant assignment in a linear consecutive k-out-of-n: G system with $n \geq 2k$. However, a corollary for the case $n < 2k$ was still incomplete. Cui and Hawkes [10] pointed out what is missing in the proof of that corollary and presented a new proof using a different approach. Cui et al. [11] studied the problems of optimal allocation between minimal and perfect repairs for a system with monotonic failure rate. For a linear and circular consecutive-k-out-of-n: F system, Yun et al. [12] analyzed the system configuration parameter k and various cost parameters. Boddu and Xing [13,14] considered the redundancy allocation problem pertaining to k-out-of-n: G heterogeneous series-parallel systems and presented a design optimization methodology based on the penalty-guided genetic algorithm. Steffen et al. [15] analyzed the reliability of actuation elements in series and parallel configurations and found the optimal configuration for a given set of requirements.

The importance measure is useful to identify the weakest component and support system reliability improvement activities. The concept of importance in reliability was first introduced by

* Corresponding author.
E-mail addresses: sisb@nwpu.edu.cn (S. Si), levitin@iec.co.il (G. Levitin), duihongyan@mail.nwpu.edu.cn (H. Dui), sdsun@nwpu.edu.cn (S. Sun).

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Nomenclature

n	number of components in the system
p_i	reliability of component i, $i=1,2,\ldots,n$
q_i	unreliability of component i, $i=1,2,\ldots,n$, $p_i+q_i=1$
$\mathbf{p}$	$(p_1,p_2,\ldots,p_n)$, component reliability vector
$(\cdot_i,\mathbf{p})$	$(p_1,\ldots,p_{i-1},\cdot_i,p_{i+1},\ldots,p_n)$
$(\cdot_i,\cdot_j,\mathbf{p}),i<j$	$(p_1,\ldots,p_{i-1},\cdot_i,p_{i+1},\ldots,p_{j-1},\cdot_j,p_{j+1},\ldots,p_n)$
$\boldsymbol{\tau}$	permutation $(\tau(1),\tau(2),\ldots,\tau(n))$ of the integers from 1 to n
$\varphi(\boldsymbol{\tau},\mathbf{p})$	system reliability corresponding to the permutation $\boldsymbol{\tau}$
$\boldsymbol{\tau}_{ij}$	permutation obtained from a permutation $\boldsymbol{\tau}$ by interchanging components in positions $\tau(i)$ and $\tau(j)$
$p_{\tau(i)}$	reliability of component residing in particular position $\tau(i)$
S	set of all possible permutations
$R(\mathbf{p})$	system reliability for component reliability vector $\mathbf{p}$
$R(n;k)$	reliability of a linear consecutive-k-out-of-n system
$Q(n;k)$	unreliability of a linear consecutive-k-out-of-n system, $R(n;k)+Q(n;k)=1$

Birnbaum [16] in 1969. The Birnbaum importance (BI) describes the contribution of the component reliability to the entire system reliability. A wide range of importance measures have been introduced since Birnbaum's work, such as Barlow–Proschan importance measure [17], joint importance measure [18], composite importance measure [19,20], etc. Si et al. [21,22] proposed the integrated importance measure of component states based on the performance for the deterioration process and based on the cost for the maintenance process. Various importance measures have been successfully applied in tackling some reliability optimization problems. The improvement potential [23], Fussell–Vesely importance [24,25], risk achievement worth [26], and risk reduction worth [26] are widely used for probabilistic safety assessment and other reliability and risk analysis problems.

The importance measures have also been used to solve the component assignment problem for consecutive-k-out-of-n system. Kuo et al. [27] discussed the relationships between the consecutive-k-out-of-n: F system and the consecutive-k-out-of-n: G system, and studied their optimal configuration by Birnbaum importance. Zuo and Kuo [4] gave a heuristic method for optimal design of a general linear consecutive-k-out-of-n system using the component Birnbaum importance. Hwang et al. [28] and Chang et al. [29,30] gave the relationships and comparisons in Birnbaum importance between the consecutive-k-out-of-n: F system and the consecutive-k-out-of-n: G system. Lin and Kuo [31] explored the relationships between reliability allocation and the Birnbaum importance for general coherent systems, and found some necessary conditions which can be used to examine whether or not a system has any invariant optimal allocation. Yao et al. [32] proposed five new Birnbaum importance-based heuristics and presented their corresponding properties. Zhu et al. [33] summarized the Birnbaum importance patterns for linear consecutive-k-out-of-n systems, and investigated the relationship between the Birnbaum importance and the common component reliability.

However, all the mentioned importance measures do not consider the possible change of system arrangement with component reliability variation. If the system structure can be easily changed (for example, by component rearrangement), then the original permutation may not be optimal after the change in the reliability of a specific component. The optimal structure can be determined for any specific combination of components' reliabilities. When the optimal system structure changes, the component importance can change as well. When the most promising component reliability improvement is determined, the possible structure change should be taken into account. This paper studies the relationships between reliability allocation and the importance by obtaining the reliability of the optimal system structure as a function of component reliability. The suggested analysis methodology is demonstrated as an example of the linear consecutive-k-out-of-n system.

The rest of the paper is organized as follows. Section 2 analyzes the system structure optimization methods for the linear consecutive-k-out-of-n system. The Birnbaum importance based on the optimal permutation is discussed in Section 3. Section 4 considers examples of linear consecutive-k-out-of-n: F and G systems to demonstrate the change of the component Birnbaum importance with the optimal system reconfiguration. The conclusions are given in Section 5.

2. System configuration optimization

All the sequence dependent systems considered in [1–4,6–8] consist of n statistically independent components arranged in a line. The objective of the system structure optimization is to assign the components to n positions on the line to maximize the reliability of the system.

Definition 1. The optimal system configuration is a permutation $\boldsymbol{\tau}^*\in S$ such that

$$\varphi(\boldsymbol{\tau}^*,\mathbf{p})=\max_{\boldsymbol{\tau}\in S}\varphi(\boldsymbol{\tau},\mathbf{p}), \tag{1}$$

where $\varphi(\boldsymbol{\tau},\mathbf{p})$ is the system reliability corresponding to the permutation $\boldsymbol{\tau}$.

When one changes the reliability of component residing in a particular position $\tau(i)$ from $p_{\tau(i)}$ to $p^*_{\tau(i)}$ and fixes the reliabilities of all other components, the system reliability becomes

$$\begin{aligned}\varphi(\boldsymbol{\tau}^*,\mathbf{p})=&\,p^*_{\tau(i)}p_{\tau(j)}R(1_i,1_j,\mathbf{p})+p^*_{\tau(i)}(1-p_{\tau(j)})R(1_i,0_j,\mathbf{p})\\&+(1-p^*_{\tau(i)})(1-p_{\tau(j)})R(0_i,0_j,\mathbf{p})+(1-p^*_{\tau(i)})p_{\tau(j)}R(0_i,1_j,\mathbf{p}).\end{aligned} \tag{2}$$

After the change of the reliability $p_{\tau(i)}$, the permutation $\boldsymbol{\tau}^*\in S$ may not become optimal. The interchange of the components initially residing in positions $\tau(i)$ and $\tau(j)$, gives the system reliability

$$\begin{aligned}\varphi(\boldsymbol{\tau}^*_{ij},\mathbf{p})=&\,p^*_{\tau(i)}p_{\tau(j)}R(1_i,1_j,\mathbf{p})+p_{\tau(j)}(1-p^*_{\tau(i)})R(1_i,0_j,\mathbf{p})\\&+(1-p^*_{\tau(i)})(1-p_{\tau(j)})R(0_i,0_j,\mathbf{p})+(1-p_{\tau(j)})p^*_{\tau(i)}R(0_i,1_j,\mathbf{p}).\end{aligned} \tag{3}$$

The variation of the system reliability after this interchange is

$$\begin{aligned}\varphi(\boldsymbol{\tau}^*,\mathbf{p})-\varphi(\boldsymbol{\tau}^*_{ij},\mathbf{p})=&\,p^*_{\tau(i)}(1-p_{\tau(j)})R(1_i,0_j,\mathbf{p})\ +(1-p^*_{\tau(i)})p_{\tau(j)}R(0_i,1_j,\mathbf{p})\\&-p_{\tau(j)}(1-p^*_{\tau(i)})R(1_i,0_j,\mathbf{p})-(1-p_{\tau(j)})p^*_{\tau(i)}R(0_i,1_j,\mathbf{p})\\=&\,(p^*_{\tau(i)}-p_{\tau(j)})R(1_i,0_j,\mathbf{p})-(p^*_{\tau(i)}-p_{\tau(j)})R(0_i,1_j,\mathbf{p})\\=&\,(p^*_{\tau(i)}-p_{\tau(j)})(R(1_i,0_j,\mathbf{p})-R(0_i,1_j,\mathbf{p})).\end{aligned} \tag{4}$$

If $\varphi(\boldsymbol{\tau}^*,\mathbf{p})-\varphi(\boldsymbol{\tau}^*_{ij},\mathbf{p})<0$, then one gets better permutation by interchanging the components in positions $\tau(i)$ and $\tau(j)$.

In a series system, $\varphi(\boldsymbol{\tau},\mathbf{p})=\prod_{j=1}^{n}p_{\tau(j)}$ and in a parallel system, $\varphi(\boldsymbol{\tau},\mathbf{p})=1-\prod_{j=1}^{n}(1-p_{\tau(j)})$. So for any permutation τ, the corresponding system reliability is the same.

In a k-out-of-n system, $R(1_i,0_j,\mathbf{p})$ and $R(0_i,1_j,\mathbf{p})$ represent the reliability of (k-1)-out-of-(n-2) system consisting of the n-2 components in positions $\tau(x), 1\le x\le n$ and $x\ne i,j$.