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Bayesian modeling of multi-state hierarchical systems with multi-level information aggregation

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ABSTRACT

Reliability modeling of multi-state hierarchical systems is challenging because of the complex system structures and imbalanced reliability information available at different system levels. This paper proposes a Bayesian multi-level information aggregation approach to model the reliability of multi-level hierarchical systems by utilizing all available reliability information throughout the system. Cascading failure dependency among components and/or sub-systems at the same level is explicitly considered. The proposed methodology can significantly improve the accuracy of system-level reliability modeling. A case study demonstrates the effectiveness of the proposed methodology.

1. Introduction

Hierarchical system structures are widely adopted in the design of complex engineering systems for its advantages of scalability, tractability and modularity [1]. A system is defined as "hierarchical" if it consists of multiple sub-systems, which may consist of multiple sub-systems/components themselves. A Hierarchical system often contains multiple levels of hierarchy. Fig. 1 shows a three-level Electro-Mechanical Actuator (EMA) system. The system consists of two sub-systems, i.e., a Motor Power Supply (PS) sub-system and an Actuator Servo Drive (ASD) sub-system. Each sub-system consists of a number of sub-systems/components, e.g., ASD sub-system consists of a Pulse-Width Modulation (PWM) Controller, an H-Bridge Circuit and a Direct Current (DC) Motor. A general hierarchical system may consist of more than three levels. In this paper, for the convenience of describing a general hierarchical system, a functional unit at the *l*th level of a system is defined as a level-*l* element, i.e., the system is defined as the level-1 element, a sub-system it consists of is called a level-2 element, and a sub-system/component of a level-2 element is called a level-3 element, etc. According to this definition, the EMA system in Fig. 1 is called a level-1 element, the sub-systems are called level-2 elements, and the sub-systems/components of a level-2 element are called level-3 elements. Such elements in a hierarchical system are interconnected and interacting with each other, jointly contributing to the whole system functionality.

In many real-world situations, performance levels or failure modes of elements may not be restricted to binary values, i.e., functioning and failed. For example, a power generating system can function at different capacity levels and partial failure of its composing elements may result in different reduced capacity levels [2]. Another example is the valves used in a fluid control system, which has two common failure modes of "stuck-open" and "stuck-closed" [3]. It is noted that a major difference between performance levels and failure modes is that the latter does not have intrinsic order. In this paper, this difference is neglected and thus they are defined interchangeably as "failure states" . A hierarchical system with multi-state elements is therefore defined as a multi-state hierarchical system (MSHS) [4]. The scope of this paper will be restricted to MSHSs. It is a subset of multi-state reliability systems and the "hierarchy" here corresponds to a system structure.

Reliability modeling of multi-state systems is challenging and traditional Boolean reliability theory is no longer appropriate. Recent years witness the development of effective approaches (e.g., stochastic process approach, universal generating function approach, etc.) in modeling multi-state system reliability, as summarized in [4]. For the MSHSs, one critical issue is the failure relationship representation. Representation of failure relationship among MSHS elements is significantly more complex than the

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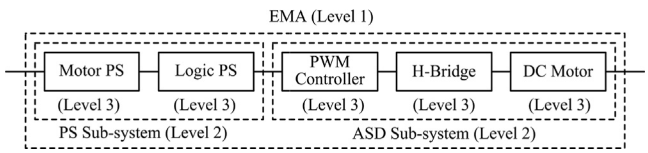

Fig. 1. Block diagram for an EMA system.

representation of typical series or parallel systems with binary failure states. The complexity is caused by the inter-level failure relationship randomness and intra-level failure dependency. Inter-level failure relationship randomness refers to the probabilistic inter-level failure relationship between elements at two adjacent levels [5]. That is, for example, given a combinatorial states of the composing sub-systems, a system's failure is not deterministic. Rather, the failure may be associated with a probability between 0 and 1. The intra-level failure dependency refers to the failure dependency among system elements at the same level. The intra-level failure dependency can be categorized into common-cause failure, cascading failure and negative dependency failure [6]. The common-cause failure relationship have been extensively studied [7,8] and the negative dependency failure is rarely encountered and can be treated similarly as cascading failure [6], which is the focus of this paper. Cascading failure is defined as the relationship that the under-performance or failure of one element in a hierarchical system will subsequently influence the performance or trigger the failure of other elements at the same level [9]. For example, in a multiprocessor system, if one sub-system, the power supply, is at its failure state of "over-voltage", it will increase the probability for the other sub-systems, processors, to break down [5]. In this paper, a Bayesian Network (BN) [10] is employed to address the aforementioned complexity of probabilistic inter-level failure relationship and cascading intra-level failure dependency. Compared to traditional representation methods, such as Fault Tree or Block Diagram, BN allows more flexible representation of failure relationship, as summarized in [10,11], and has been successfully applied in many multi-state engineering systems [12–14].

Another critical issue in the reliability engineering modeling of MSHSs is the imbalanced reliability information available for elements at different levels. Reliability information is defined as the reliability test data and prior knowledge for elements. Prior knowledge includes historical studies (e.g., past reliability test results, warranty and maintenance records, etc.) and domain knowledge (e.g., expert's judgment, engineering experience, etc.). With the development of reliability data collecting technology and information storage systems, reliability information for elements closer to the bottom of the hierarchy of an MSHS, e.g., components, is either abundantly available or easily accessible. This is because these elements are often selected from standard products with high volume of production and deployment. Comprehensive knowledge and empirical data on failures can be accumulated from operations, maintenance, and inexpensive reliability tests. However, reliability information is often scarce or even absent for elements closer to the top of the hierarchy, especially for the system as a whole, since reliability tests are often costly and time-consuming and accumulated knowledge is limited. Therefore, to estimate the reliability of an MSHS, it is desirable to integrate the available reliability information from elements at all levels. In the existing literature, Bayesian approaches are a popular choice due to the capability in multi-source data fusion [15,16]. For example, Martz et al. [17,18] proposed a multi-level information aggregation approach for series and parallel systems with binary failure states; Hulting and Robinson [19] extended the method to failure-time data in series system structures; Li et al. [20] proposed a semi-parametric modeling approach for hierarchical systems with multi-level information aggregation; Johnson et al. [21] present a "full-Bayesian approach" to integrate all reliability information of the system. Recent work was well summarized in [22,23]. However, most of the existing work is based on the assumption of independent failure relationship and, thus, cannot be applied to MSHS with inter-level probabilistic failure relationship and intra-level cascading failures.

To fill out the gap of existing research and provide a generic information aggregation framework, a Bayesian multi-level information aggregation approach is proposed in this paper to estimate the system reliability of an MSHS. The proposed method models system reliability by simultaneously considering: (i) *multiple failure states of an element*; (ii) *inter-level probabilistic failure relationship*; (iii) *intra-level cascading failure dependency*. The rest of this paper is organized as follows: Section 2 presents the BN representation of the failure relationship among elements in an MSHS. Section 3 gives a detailed illustration of the proposed multi-level information aggregation method. A numerical case study presented in Section 4 demonstrates the effectiveness of the proposed method and Section 5 concludes the paper.

2. BN representation of MSHS

In this paper, a BN is employed to represent the system structure of an MSHS. As compared to traditional methods, such as Fault Tree, BN possesses the following attractive features in representing failure relationship of an MSHS: (i) Fault Tree mainly represents elements with binary failure states, whereas BN could deal with multiple failure states; (ii) Logic gates in Fault Tree, such as "AND" gate and "OR" gate, can only represent simple, deterministic inter-level failure relationship, whereas conditional probability tables in BN could represent complex, probabilistic relationship with deterministic relationship being just a special case (i.e., probability 0 or 1); (iii) Fault tree represents elements at the same level with independence assumption, whereas BN releases such assumption by allowing intra-level cascading dependency among elements.

A general BN consists of nodes, $\{X_1, \ldots, X_n\}$, and directed arcs between some nodes. Fig. 2 shows a BN with 5 nodes and 7 arcs [24,25]. Each node X_i is a random variable. If there is a directed arc from X_i to X_j, X_i is called a "parent" of X_j . There may be no or more than one parents for each node. An arc characterizes the probabilistic dependency of a node on its parent nodes. That is, depending on the values a node's parents take on, the conditional probability distribution of the node may be different. In this paper, a BN is employed to represent the cause-and-effect failure relationship among elements of an MSHS, in which nodes are elements of the MSHS. Since elements are at different levels, a

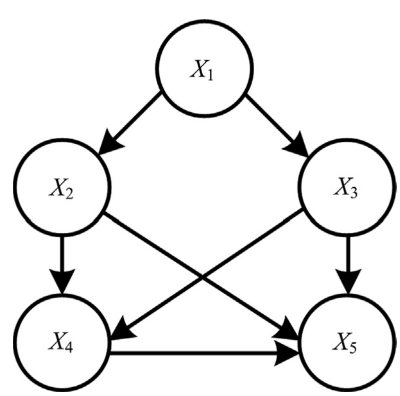

Fig. 2. A general BN structure.