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Matrix stochastic analysis of the maintainability of a machine under shocks

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ABSTRACT

We study the maintenance of a machine operating under environmental conditions producing shocks affecting the lifetime of the machine. The shocks cause different types of damage depending on their strength and eventually the total failure. The maintenance of the machine is performed by repairs and replacement. The interarrival times of shocks are dependent. We introduce a multidimensional stochastic model for simulating the evolution of the lifetime of the machine. This model implies the application of the matrix-analytic methods, that are being used in stochastic modelling with interesting results. Under this methodology, the availability, the reliability, and the rates of occurrence of the different types of failures and of the replacements are calculated, obtaining mathematically tractable expressions. The results are applied to a numerical example.

1. Introduction

Environmental conditions are often the main cause of failure in certain type of systems. In the literature it is usual to denote by shocks the influence of the external conditions on the lifetime and deterioration of the systems. The term shock is considered in a wide sense, it can be voltage, vibrations, strength, and others. These shocks affect the systems provoking the failure or the deterioration, depending on their strength. This is the case of the systems operating in extreme conditions; for example, systems under very high strength that fail mainly due to this cause; motors under vibrations of high intensity as the main source of failures, and others.

The probabilistic models for representing shock and wear systems are based on stochastic processes, these structures allow to describe the evolution of the system with time. The first paper related to shock and wear models [6] presented a device subject to shocks. In this paper the shocks arrive following a homogeneous Poisson process and producing damage on the device. The methodology for studying the survival function is non-parametric. From this paper, many others have been written about shock and wear models following different methodologies. Explicit expressions for reliability function and other reliability measures in shock models under arrival processes simpler than the one we present are calculated in recent papers [8,4,5]. The first paper including matrix-analytic methods in the study of shock and wear models is [21]. These methods were presented in [19], and an analysis of them with applications on different domains has been done in [1]. In [9,17], they are applied to the study of systems subject to shocks considering phase-type distributions. The inclusion of the Markovian arrival processes has contributed to a more ample study of the stochastic models, they are frequently used in queueing theory and have been applied in shocks and wear systems in [12,17,16,22]. In [18] a system under two independent types of failure: internal, governed by phase-type distribution, and external, governed by a Markovian arrival process is studied. In [15] it is proved that the matrix-analytic methods are a versatile procedure for introducing different types of repair (perfect, imperfect and improving) in the general study of a shock and wear system and it can be extended to other reliability systems. Many replacement policies on shock models have been done by [10,23,11,7] and many others. In general, in the different models that have been considered in the literature the occurrence of failures due to the shocks and the ones due to internal failures are independent.

The present paper considers a stochastic process as the source of the shocks causing damage to a machine, completing these previous papers. The shocks cause failure on the internal and external structures of the machine. Depending on the strength of the shocks two types of failures are considered: those needing a time to complete the repair and others that can be repaired in a negligible time. So two types of repair are considered: perfect and minimal. It is also taken into account that the arrival of fatal shocks

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causes the failure of the machine without the possibility of repair, then it is replaced by another new and identical one. The random cumulation of shocks that can be minimally repaired can cause a non-repairable failure and consequently the replacement of the system. The arrival of shocks and the repair times are random. A model for simulating the arrival of so many different types of failures and the maintenance of the system is a complex one. We applied the matrix-analytic methods for studying a system under these conditions.

It will be assumed that the arrival of shocks follows a Markovian arrival process; the perfect repairs follow a phase-type distribution, and the time of minimal repair is negligible. After a random finite number of minimal repairs the machine is replaced, this number is governed by a discrete phase-type distribution. The time of the replacement is instantaneous. This is a general system and contributes to the study of shock and wear systems in several ways: (1) The arrival of shocks extends the usual arrival processes used in the literature, such as the Poisson process, Poisson-modulated-Markov process, and phase-type renewal process, among others. An important property of this process is that the interarrival times between different types of shocks are dependent. (2) The phase-type distributions are a class of distributions dense in the family of distributions defined on the positive real half-line. This means that any distribution function can be approximated by a distribution of this class. (3) There are several types of failures, internal and external, repairable and non-repairable. (4) The study is performed considering the evolution of the machine in finite time (transient regime) and the one in the long run (stationary regime).

When matrix-analytic methods are considered the system under study is governed by a multidimensional Markov process. Using the Markovian methodology we calculate the availability, the reliability, the mean number of minimal repairs, the mean number of perfect repairs and the mean number of replacements per unit time in transient and stationary regime. The system, the calculation of the performance measures, and the applied methodology are contributions of the paper to the study of the reliability systems. The expressions calculated following these methods are presented in a tractable form allowing an algorithmic treatment and the computational calculations in order to reach numerical expressions of the main quantities associated to the reliability of the system.

The paper is organized as follows. In Section 2 the Markov model governing the general system is constructed. In Section 3 the system is studied under transient regime. In Section 4 the renewal phase-type process modeling the replacements is calculated. In Section 5 the system is studied under stationary regime. In Section 6 a numerical application is performed. Some conclusions are given at the end of the paper.

The phase-type distributions, the Markovian arrival process, and the matrix operations of Kronecker play an important role in the present paper. We define them formally for a better comprehension of the paper. For more details see [20,3].

Definition 1. The distribution function $H(\cdot)$ on $[0,\infty[$ of a phase-type distribution (PH-distribution) is

$$H(x) = 1 - \alpha \exp(Tx)e, \quad x \geq 0.$$

It is associated to a finite Markov process with one absorbent state. The initial vector of the process is α. Matrix T is the submatrix of the generator of the process restricted to the transient states. Vector e is a column vector of 1's of appropriate order. The absorption column vector is denoted by T^0 and it satisfies $Te + T^0 = 0$. The order of the vectors and matrices involved are the same and it is the order of the distribution. It is said that the distribution has representation (α, T) and it is written $PH(\alpha, T)$.

Definition 2. The probability density of a discrete phase-type is

$$p_k = \beta S^{k-1} S^0, \quad k \geq 1.$$

It is associated to a finite Markov chain with one absorbent state. The initial vector of the chain is β. Matrix S is the submatrix of the transition matrix of the process restricted to the transient states. Vector e is a column vector of 1's of appropriate order. The absorption column vector is denoted by S^0 and it satisfies $Se + S^0 = e$. The order of the vectors and matrices involved are the same, and it is the order of the distribution. It is said that the distribution has representation (β, S) and it is written $PH_d(\beta, S)$.

Definition 3. Let D be an irreducible infinitesimal generator of a Markov process. Let a sequence of matrices $D_k, k \geq 1$ be non-negative and the matrix D_0 with non-negative off-diagonal entries. The diagonal entries of D_0 are strictly negative and it is non-singular. It satisfies

$$D = D_0 + \sum_{k \geq 1} D_k.$$

All the matrices are square and have the same order. Associated to this Markov process there is a renewal Markov process performing an arrival process to the real line operating as follows. Matrix D_0 governs the interarrival times and matrix D_k governs the arrival of type k, $k \geq 1$. The initial vector of the renewal Markov process is denoted by d. This is the Markovian arrival process (MAP) associated to the initial Markov process. The order of the MAP is the order of the involved matrices.

Definition 4. If A and B are rectangular matrices of orders $m_1 \times m_2$ and $n_1 \times n_2$, respectively, their Kronecker product $A \otimes B$ is the matrix of order $m_1 n_1 \times m_2 n_2$, written in compact form as $(a_{ij} B)$.

The Kronecker sum of the square matrices C and D of orders p and q, respectively, is defined by $C \oplus D = C \otimes I_q + I_p \otimes D$, where I_k denotes the identity matrix of order k.

2. The model

We consider a machine subject to shocks producing internal and external damages. The assumptions of the system about how the random arrival of failures occurs, what types of damage are produced in the machine, which are the sojourn times in repair, and the random number of minimal repairs that the system can stand are the following:

Assumption 1. There is only a source of failures governed by an MAP. These failures cause damage in the machine.

Assumption 2. Two different types of failure are produced in the machine by the shocks: internal and external. These types of failure can be repairable or non-repairable.

Assumption 3. Two types of repair are performed, depending on the strength of the shock: perfect and minimal.

Assumption 4. The minimal repair is instantaneous. After the repair the machine is as bad as old.

Assumption 5. The time of perfect repair follows a phase-type distribution. After the repair the machine is as good as new.